

Unit 5: Gravitation

Textbook Exercise Questions

5.1 Encircle the correct answer from the given choices.

i. Earth's gravitational force of attraction vanishes at:

- (a) 6400 km (b) infinity ✓
(c) 42300 km (d) 1000 km

ii. Value of g increases with the:

- (a) Increase in mass of body (b) increase in altitude
(c) decrease in altitude ✓ (d) none of the above

iii. The value of g at a height one Earth's radius above the surface of Earth is:

- (a) $2g$ (b) $\frac{1}{2}g$
(c) $\frac{1}{3}g$ (d) $\frac{1}{4}g$ ✓

iv. The value of g on moon's surface is 1.6 ms^{-2} . What will be the weight of a 100 kg body on the surface of the moon?

- (a) 100 N (b) 160 N ✓
(c) 1000 N (d) 1600 N

v. The altitude of geostationary orbits in which communications satellites are launched above the surface of Earth is:

- (a) 850 km (b) 1000 km
(c) 6400 km (d) 42300 km ✓

vi. The orbital speed of a low orbit satellite is:

- (a) zero (b) 8 ms^{-1}
(c) 800 ms^{-1} (d) 8000 ms^{-1} ✓

5.2 What is meant by force of gravitation?

Ans: In the universe, there exists a force between the bodies due to which everybody of the universe attracts every other body. This force is known as force of gravitation.

5.3 Do you attract the earth or the Earth attracts you? Which one is attracting with a larger force? You or Earth?

Ans: We attract the earth and Earth attracts us but Earth attracts us with larger force because the mass of the Earth is large.

5.4 What is a field force?

Ans: The gravitational pull of the Earth acting on the body whether the body is in contact with the Earth or not is called field force.

5.5 Why earlier scientists could not guess about the gravitational force?

Ans: The early scientists could not guess about the gravitational force due to lack of observations and lack of knowledge. Also it is a very weak force and its presence cannot be detected until mass of one body is much greater than mass of other body.

5.6 How can you say that gravitational force is a field force?

Ans: The gravitational force exists around the Earth and is acting on the bodies whether the bodies are in contact with the Earth or not. So, we can say that gravitational force is a field force.

5.7 Explain, what is meant by gravitational field strength? (LHR 2013)

Ans: In gravitational field, the gravitational force acting per unit mass is called the gravitational field strength. It becomes weaker and weaker as we go away from the object applying the gravitational force.

5.8 Why law of gravitation is important to us?

Ans: Law of gravitation is important to us because it is used to calculate force of attraction between two masses. It is used to calculate the mass of Earth.

5.9 Explain the law of gravitation?

Ans: See Q. no.1 Long Question

5.10 How the mass of Earth can be determined?

Ans: See Q. no.3 Long Question

5.11 Can you determine the mass of our moon? If yes, then what you need to know?

Ans: Yes we can determine the mass of the moon by same method used to measure the mass of the Earth with the help of law of gravitation. The formula is:

$$M_m = \frac{g_m R_m^2}{G}$$

From the above relation it shows that we require,

g_m = gravitational acceleration on the surface of moon

R_m = Radius of moon

G = Gravitational constant

5.12 Why does the value of g vary from place to place? (GRW 2015, LHR 2016)

Ans: We know that

$$g = \frac{GM_e}{R^2}$$

The above relation shows that value of 'g' is inversely proportional to the square of distance of body from the centre of earth. Hence when distance body from centre of earth increase, the value of g decreases and vice versa.

5.13 Explain how the value of g varies with altitude.

Ans: See Q. no.4 Long Question

5.14 What are artificial satellites? (LHR 2013)

Ans: Scientists have sent many objects into space. Some of these revolve around the Earth. These are called artificial satellites.

Large numbers of artificial satellites have been launched in different orbits around the Earth. They take different time to complete their one revolution around the Earth depending upon their distance h from the Earth.

5.15 How Newton's law of gravitation helps in understanding the motion of satellites.

Ans: When a satellite moves around the earth in a nearly circular path, the gravitational force of attraction between earth and satellite provides the necessary centripetal force for its motion. This gravitational force can be found by using Newton's law of gravitation and finally we can find orbital speed of satellites.

5.16 On what factors the orbital speed of a satellite depends? (GRW 2015, LHR 2016)

Ans: As we know that

$$v_o = \sqrt{g_h (R + h)}$$

So, we can say that orbital speed depends upon the gravitational acceleration and distance between the center of earth and the satellite.

5.17 Why communication satellites are stationed at geostationary orbits? (GRW 2013, 14)

Ans: The satellites in geostationary orbits remain all the time in front of target part of Earth so that direction of receiver's dish do not to be changed.

Unit 5: Gravitation

Long Questions

5.1 THE FORCE OF GRAVITATION

Law of Gravitation

Q.1 State and explain Newton's law of gravitation. (GRW 2011, 12, 13, 15, LHR 2013)

Ans: Gravitation

In the universe, there exists a force between the bodies due to which everybody of the universe attracts every other body. This force is known as force of gravitation.

Statement

Every object in the universe attracts every other object with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centers.

Explanation

Every object in this universe attracts other objects towards its centre. The attraction between two objects is called gravitation. On the basis of his observations, Newton derived a law which is called Newton's law of gravitation.

Mathematical Derivation

Consider two bodies A and B of masses m_1 , and m_2 , respectively. According to law of gravitation, the gravitational force of attraction F with which two mass m_1 and m_2 separated by a distance d attracts each other is given by:

$$F \propto m_1 m_2$$

$$F \propto \frac{1}{d^2}$$

OR
$$F \propto \frac{m_1 m_2}{d^2}$$

$$F = \frac{G m_1 m_2}{d^2}$$

Gravitational constant

G is a constant called gravitational constant. It is called universal constant of gravitation. If $m_1 = m_2 = 1$ kg and $d = 1$ m, then $F = G$. Thus G is a force which 1 kg object exerts on another 1 kg object placed 1 m away from it. In SI units, the value of gravitational constant G is $6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$.

Dependence of Gravitational force on mass

Due to small value of G , the gravitational force of attraction between different objects around us is very small, so we do not feel it. However, if the mass of one or both the objects is very large, then we can observe the effect of gravitational force easily.

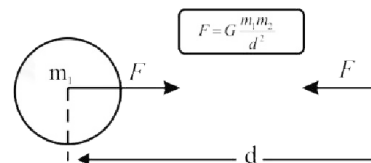


Figure 5.1: Two masses attract each other with a gravitational force of equal magnitude.



Figure 5.2: Weight of a body is due to the gravitational force between the body and the Earth.

Q.2 Explain the gravitational field?**(GRW 2013)**

Ans: The space around the earth in which its gravitational force acts on a body is called gravitational field. According to the Newton's law of gravitation, the gravitational force between a body of mass m and the Earth is given by,

$$F = \frac{G m M_e}{r^2}$$

Where M_e is the mass of the Earth and r is the distance of the body from the centre of the Earth.

The weight of a body is due to the gravitational force with which Earth attracts a body. Gravitational force is a non-contact force.

Example

The velocity of a body, thrown up, goes on decreasing while on returns its velocity goes on increasing. This is due to the gravitational pull of the Earth acting on the body whether the body is in contact with the Earth or not. Such a force is called the field force. It is assumed that a gravitational field exists all around the Earth. This field is directed towards the centre of the Earth. The gravitational field becomes weaker and weaker as we go farther and farther away from the Earth.

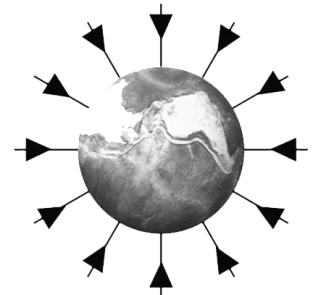


Figure 5.3: Gravitational field around the Earth is towards its centre.

Gravitational Field Strength

In the gravitational field of the Earth, the gravitational force per unit mass is called gravitational field strength of the Earth. At any place its value is equal to the value of g at that point. Near the surface of the Earth, the gravitational field strength is 10 Nkg^{-1} .

Q.3 Determine the mass of the earth by using Newton's law of gravitation.**(GRW 2014, LHR 2015)**

Ans: Suppose a body of mass m is placed on the surface of the Earth. Let mass of the Earth is M_e and radius of Earth be R . The distance between the body and centre of the Earth is equal to the radius of the Earth R .

According to the law of gravitation, the gravitational force F of the Earth acting on the body is given by,

$$F = \frac{G M_e m}{R^2} \dots\dots\dots (1)$$

We know that the force of gravitation with which Earth attracts the body towards its centre is equal to the weight of the body. Therefore,

Therefore, $F = w = mg$

OR $mg = \frac{G M_e m}{R^2}$

Or $g = \frac{G M_e}{R^2} \dots\dots\dots (2)$

Or $M_e = \frac{g R^2}{G} \dots\dots\dots (3)$

As we know that,

$$g = 10 \text{ ms}^{-2}$$

$$R = 6.4 \times 10^6 \text{ m}$$

And $G = 6.673 \times 10^{-11} \text{ Nm}^2\text{kg}^{-2}$

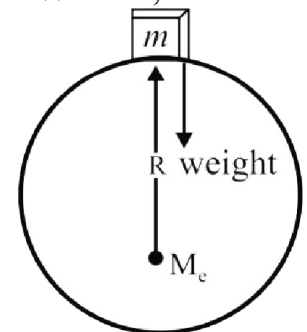


Figure 5.4: Weight of a body is Equal to the gravitational force between the body and the Earth

By putting the value of g , R and G in equation (3), we have

$$M = \frac{gR^2}{G} = \frac{10 \times (6.4 \times 10^6)^2}{6.673 \times 10^{-11}}$$

$$M = \frac{10 \times 40.96 \times 10^{12}}{6.673 \times 10^{-11}}$$

$$M = \frac{409.6 \times 10^{12}}{6.673 \times 10^{-11}}$$

$$M = 61.4 \times 10^{23}$$

$$M = 6.14 \times 10^{24} \text{ kg}$$

$$M = 6 \times 10^{24} \text{ kg}$$

Hence the mass of the earth is approximately **$6 \times 10^{24} \text{ kg}$** .

Q.4 Explain the variation of 'g' with altitude.

Ans: As we know that

$$g = \frac{G M_e}{R^2}$$

The above equation show that the acceleration due to gravity depends on the radius of Earth at its surface. The value of g is inversely proportional to the square of the radius of the Earth. It does not remain constant. It decreases with altitude. Altitude is the height of an object or place above sea level. The value of g is greater at sea level than at the hills.

(GRW 2015)

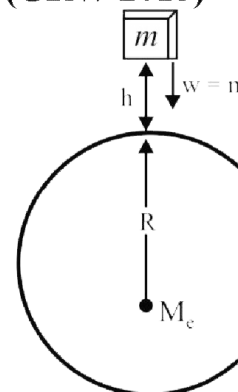


Figure 5.5: Weight of a body decreases as its height increases from the surface of the Earth.

Mathematical Form

Suppose a body of mass m at an altitude h . The distance of the body from the centre of the Earth is $R+h$. By using above equation, we have

$$g_h = \frac{G M}{(R + h)^2}$$

According to the above equation, we come to know that at a height equal to one Earth radius above the surface of the Earth, g becomes one fourth of its value on the Earth. Similarly, at a distance of two Earth's radius above the Earth's surface, the value of g becomes one ninth of its value on the Earth.

Q.5 What are artificial satellites? Define orbital velocities and what do you know about communication satellites? (LHR 2013)

Ans: An object that revolves around a planet is called a satellite.

Orbital Velocity

It is the velocity of the satellite with which it moves around the earth at specific height.

Natural satellite of Earth

The moon revolves around the Earth so moon is the natural satellite of Earth.

Artificial satellites

Scientists have sent many objects into space. Some of these revolve around the Earth. These are called artificial satellites.

Most of the artificial satellites orbiting around the Earth are used for communication purposes. Artificial satellites carry instruments or passengers to perform experiments in the space.

Large numbers of artificial satellites have been launched in different orbits around the Earth. They take different time to complete their one revolution around the Earth depending upon their distance h from the Earth.

Communication Satellites

Communication satellites take 24 hours to complete their one revolution around the Earth. As Earth also completes one rotation about its axis in 24 hours, hence, these communication satellites appear to be stationary with respect to Earth. It is due to this reason that the orbit of such satellites is called geostationary orbit. Dish antennas sending and receiving the signals from them have fixed direction depending upon their location on the Earth.

Q.6 Explain the motion of an artificial satellite and derive the formula for orbital velocity of an artificial satellite. (LHR 2013, 2014, 2015, GRW 2014)

Ans: A satellite requires centripetal force that keeps it to move around the Earth. The gravitational force of attraction between the satellite and the Earth provides the necessary centripetal force.

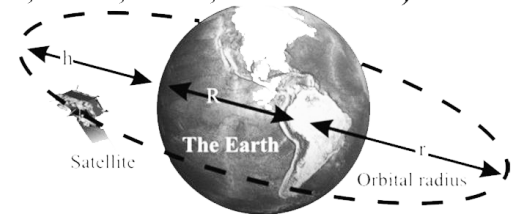


Figure 5.6: A satellite is orbiting around the Earth at a height h above the surface of the Earth.

Mathematical Derivation

Suppose a satellite of mass m is revolving around the Earth at a height ' h ' in an orbit of radius r_o with orbital velocity v_o . The necessary centripetal force F_c required to keep the satellite moving is given by,

$$F_c = \frac{mv_o^2}{r_o} \dots\dots\dots (1)$$

This centripetal force is provided to the satellite by the gravitational force of attraction between the Earth and satellite and is equal to the weight of the satellite w (mg_h). thus

$$F_c = w = mg_h \dots\dots\dots (2)$$

By comparing equation (1) and equation (2), we get

$$\text{Or } mg_h = \frac{mv_o^2}{r_o}$$

$$\text{Or } v_o^2 = g_h r_o$$

$$\text{Or } v_o = \sqrt{g_h r_o}$$

$$\text{As } r_o = R + h$$

$$\text{So } v_o = \sqrt{g_h (R + h)} \dots\dots\dots (3)$$

This equation represents the orbital velocity, which a satellite must possess when launched in an orbit of radius $r_o = R + h$ around the Earth. An approximation can be made for a satellite revolving close to the Earth such that $R \gg h$.

$$R + h \approx R$$

$$\text{And } g_h \approx g$$

$$\text{So } v_o = \sqrt{g R}$$

A Satellite revolving around very close to the Earth has speed nearly 8 kms^{-1} or 29000 kmh^{-1} .

Unit 5: Gravitation

Multiple Choice Questions

1. ----- predicted about artificial satellites about 300 years ago.
(a) Galileo (b) Newton
(c) Einstein (d) Faraday
2. Unit of gravitational field strength is:
(a) N (b) N kg^{-1}
(c) J (d) N m
3. Distance of moon from Earth is? (GRW 2013, 2014)
(a) 38, 000 km (b) 3, 80, 000 km
(c) 3, 000, 000 km (d) 30, 000 km
4. Speed of GPS satellite is:
(a) 7.9 kms^{-1} (b) 3.87 kms^{-1}
(c) 5.6 kms^{-1} (d) 5.0 kms^{-1}
5. If the distance between two masses is half then the force of gravitation becomes:
(a) One fourth (b) Four times
(c) Doubled (d) Half
6. In System International, the value of G is: (GRW 2012)
(a) $6.4 \times 10^6 \text{ Nm}^2\text{kg}^{-2}$ (b) $6.4 \times 10^{-11} \text{ Nm}^2\text{kg}^{-2}$
(c) $6.67 \times 10^{11} \text{ Nm}^2\text{kg}^{-2}$ (d) $6.67 \times 10^{-11} \text{ Nm}^2\text{kg}^{-2}$
7. Radius of earth is:
(a) $6.4 \times 10^6 \text{ km}$ (b) $6.4 \times 10^6 \text{ m}$
(c) $6 \times 10^6 \text{ m}$ (d) $6 \times 10^6 \text{ km}$
8. The SI unit of gravitational force is:
(a) $\text{Nm}^2\text{kg}^{-2}$ (b) Newton
(c) ms^{-2} (d) both “a” and “b”
9. What will be the value of G if mass of the earth becomes four times:
(a) No change (b) Four times
(c) One fourth (d) Doubled
10. The mass of Earth is approximately:
(a) $6.9 \times 10^{24} \text{ kg}$ (b) $6.0 \times 10^{24} \text{ kg}$
(c) $6.0 \times 10^{24} \text{ kg}$ (d) $5500 \times 10^{24} \text{ kg}$
11. As we go up the value of G becomes:
(a) Unchanged (b) Increases
(c) Decreases (d) Doubled
12. The force which pulls the object towards the center of circle is known as ----- force:
(a) Frictional (b) Coulomb
(c) Centripetal (d) Gravitational
13. When an object is at a height equal to radius of earth above the surface of the earth. What is the value of g_h ? (LHR 2013)
(a) $4g$ (b) $2g$
(c) $g/2$ (d) $g/4$
14. What is not true about g?
(a) g is different at different places (b) g is greater at poles
(c) g is less at poles (d) g decrease as go higher

15. If the weight of an object on the surface of earth is W . Its weight on the surface of moon will be:
 (a) $6W$ (b) $W/6$
 (c) $W/4$ (d) $W/8$
16. On mountains our weight will be ----- as compared to weight on the surface of earth.
 (a) Equal (b) Greater
 (c) Less (d) None of above
17. If mass of both the bodies is 1kg and distance between their centers is 1m then the gravitational force will be equal to:
 (a) G (b) g
 (c) V (d) None of above
18. A satellite is revolving around the earth in a circular orbit. If the radius of the orbit is increased from R to $2R$. What will be its velocity?
 (a) $\sqrt{2}v$ (b) v^2
 (c) $v/2$ (d) $\frac{v}{\sqrt{2}}$
19. An artificial satellite keeps on revolving around the earth in different orbits with uniform speed due to the?
 (a) Gravitational force (b) Frictional force
 (c) Coulmb force (d) Electromagnetic force
20. Relative velocity of Geostationary satellite with respect to earth is:
 (a) 7.9 kms^{-1} (b) 11.2 kms^{-1}
 (c) 9.8 ms^{-1} (d) Zero
21. If a rocket is fired vertically with a speed of -----, it will start revolving around the earth:
 (a) 8 ms^{-1} (b) 8 kms^{-1}
 (c) 9.8 ms^{-1} (d) 11.2 kms^{-1}
 (GRW 2013, LHR 2015)
22. Height of the Geostationary satellite above the surface of earth is:
 (a) 1000 km (b) 3600 km
 (c) 36000 km (d) 42300 km
23. Gravitational force on the surface of earth is equal to:
 (a) G (b) g
 (c) W (d) All of above
24. Weight of the body of mass 10 kg on the surface of moon: (LHR 2016)
 (a) 160 N (b) 16N
 (c) 1.62 N (d) None of above

ANSWER KEY

Q.	Ans	Q.	Ans	Q.	Ans
1	b	11	a	21	b
2	b	12	c	22	d
3	b	13	d	23	c
4	b	14	c	24	b

5	b	15	b	
6	d	16	c	
7	b	17	a	
8	b	18	a	
9	a	19	a	
10	c	20	d	

Unit 5: Gravitation

Problems

5.1 Find the gravitational force of attraction between two spheres each of mass 1000 kg. The distance between the centers of the spheres is 0.5m.

Given Data

Mass of each sphere = $m_1 = m_2 = 1000 \text{ kg}$

Distance between their centers = $d = 0.5 \text{ m}$

Required

Gravitational force between the spheres = $F = ?$

Solution

From the law of gravitation, we have

$$F = \frac{G m_1 m_2}{d^2}$$

By putting the values, we have

$$F = \frac{6.67 \times 10^{-11} \times 1000 \times 1000}{(0.5)^2}$$

$$F = \frac{6.67 \times 10^{-5}}{0.25}$$

$$F = 26.68 \times 10^{-5}$$

$$F = 2.67 \times 10^{-4} \text{ N}$$

Result

Gravitational force between the spheres = $F = 2.67 \times 10^{-4} \text{ N}$

5.2 The gravitational force between two identical lead spheres kept at 1 m apart is 0.006673 N. Find their masses.

Given Data

Gravitational force = $F = 0.006673 \text{ N}$

Distance between centers = $r = 1 \text{ m}$

Gravitational constant = $6.67 \times 10^{-11} \text{ Nm}^2\text{kg}^{-2}$

Required

Mass of each lead spheres = $m_1 = m_2 = ?$

Solution

From law of gravitation, we have

$$F = G \frac{m_1 \times m_2}{r^2}$$

$$\text{OR } m_1 \times m_2 = \frac{F \times r^2}{G}$$

By putting the values, we have

$$m_1 \times m_2 = \frac{0.006673 \times (1)^2}{6.67 \times 10^{-11}}$$

$$m_1 \times m_2 = 0.001000 \times 10^{11}$$

$$m_1 \times m_2 = 1.00 \times 10^8$$

As $m_1 = m_2$

So $m_1^2 = 1.00 \times 10^8$

$$m_1 = 1.00 \times 10^4 \text{ kg}$$

So $m_2 = 1.00 \times 10^4 \text{ kg}$

Result

$$\text{Mass of each lead spheres} = m_1 = m_2 = 1 \times 10^4 \text{ kg}$$

5.3 Find the acceleration due to gravity on the surface of the Mars. The mass of Mars is $6.42 \times 10^{23} \text{ kg}$ and its radius is 3370 km.

Given Data

$$\text{Mass of the mars} = M = 6.42 \times 10^{23} \text{ kg}$$

$$\text{Radius of mars} = R = 3370 \text{ km} = 3370 \times 10^3 \text{ m} = 3.37 \times 10^6 \text{ m}$$

Required

$$\text{Gravitational acceleration} = g = ?$$

Solution

As we know that

$$g = \frac{GM}{R^2}$$

by putting the values, we have

$$g = \frac{6.67 \times 10^{-11} \times 6.42 \times 10^{23}}{(3.77 \times 10^6)^2}$$

$$g = \frac{42.8214 \times 10^{12}}{11.3569 \times 10^{12}}$$

$$g = 3.77 \text{ ms}^{-2}$$

Result

$$\text{Gravitational acceleration} = g = 3.77 \text{ ms}^{-2}$$

5.4 The acceleration due to gravity on the surface of moon is 1.62 ms^{-2} . The radius of Moon is 1740 km. Find the mass of moon.

Given Data

$$\text{Gravitational acceleration on Moon} = g_m = 1.62 \text{ ms}^{-2}$$

$$\text{Radius of moon} = R_m = 1740 \text{ km} = 1740 \times 10^3 \text{ m} = 1.74 \times 10^6 \text{ m}$$

$$\text{Gravitational constant} = G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$$

Required

$$\text{Mass of the moon} = M = ?$$

Solution

As we know that

$$M = \frac{gR^2}{G}$$

by putting the values, we have

$$M = \frac{1.62 \times (1.74 \times 10^6)^2}{6.67 \times 10^{-11}}$$

$$M = \frac{1.62 \times 3.0276 \times 10^{12}}{6.67 \times 10^{-11}}$$

$$M = \frac{4.90 \times 10^{12}}{6.67 \times 10^{-11}}$$

$$M = 0.735 \times 10^{23}$$

$$M = 7.35 \times 10^{22} \text{ kg}$$

Result

Mass of the moon = $M = 7.35 \times 10^{22} \text{ kg}$

5.5 Calculate the value of g at a height of 3600 km above the surface of the Earth.

Given Data

Height above the surface of Earth = $h = 3600 \text{ km} = 3600 \times 10^3 = 3.6 \times 10^6 \text{ m}$

Gravitational constant = $G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$

Mass of Earth = $M = 6 \times 10^{24} \text{ kg}$

Required

Gravitational acceleration = $g = ?$

Solution

As we know that

$$g = \frac{GM}{(R+h)^2}$$

By putting the values, we have

$$g = \frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{(6.4 \times 10^6 + 3.6 \times 10^6)^2}$$

$$g = \frac{40.02 \times 10^{13}}{(10 \times 10^6)^2}$$

$$g = \frac{40.02 \times 10^{13}}{1 \times 10^{14}}$$

$$g = 40.02 \times 10^{-1}$$

$$g = 4.002 \text{ ms}^{-2}$$

$$g = 4.0 \text{ ms}^{-2}$$

Result

Gravitational acceleration = $g = 4 \text{ ms}^{-2}$

5.6 Find the value of g due to the Earth at geostationary satellite. The radius of the geostationary orbit is 48700 km.

Given Data

Radius of geostationary satellite = $R = 48700 \text{ km} = 48700 \times 10^3 \text{ m} = 4.87 \times 10^7 \text{ m}$

Mass of earth = $M = 6 \times 10^{24} \text{ kg}$

Gravitational constant = $G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$

Required

Gravitational acceleration = $g_h = ?$

Solution

As we know that

$$g = \frac{GM}{(R+h)^2}$$

By putting the values, we have

$$g = \frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{(4.87 \times 10^7)^2}$$

$$g = \frac{40.02 \times 10^{13}}{23.72 \times 10^{14}}$$

$$g = 1.68 \times 10^{-1}$$

$$g = 0.168 \text{ ms}^{-2}$$

$$g = 0.17 \text{ ms}^{-2}$$

Result

Gravitational acceleration = $g_h = 0.17 \text{ ms}^{-2}$

5.7 The value of g is 4.0 ms^{-2} at a distance of 10000 km from the centre of the Earth. Find the mass of the Earth.

Given Data

Gravitational acceleration = $g_h = 4.0 \text{ ms}^{-2}$

Distance from centre of Earth = $R+h = 10000 \text{ km} = 10000 \text{ km} = 1 \times 10^7 \text{ m}$

Gravitational constant = $G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$

Required

Mass of earth = $M = ?$

Solution

As we know that

$$g_h = \frac{GM_e}{(R+h)^2}$$

By putting the values, we have

$$M_e = \frac{g_h (R+h)^2}{G}$$

$$M_e = \frac{4 \times (1.0 \times 10^7)^2}{6.67 \times 10^{-11}}$$

$$M_e = \frac{4 \times 10^{14}}{6.67 \times 10^{-11}}$$

$$M = 0.599 \times 10^{25}$$

$$M_e = 5.99 \times 10^{24}$$

$$M_e = 6 \times 10^{24} \text{ kg}$$

Result

Mass of Earth = $M_e = 6 \times 10^{24} \text{ kg}$

5.8 At what altitude the value of g would become one fourth than on the surface of the Earth?

Given Data

Gravitational acceleration = $g = 10 \text{ m}^{-2}$

Gravitational acceleration at height = $g_h = \frac{g}{4} = \frac{10}{4} = 0.25 \text{ ms}^{-2}$

Gravitational constant = $G = 6.67 \times 10^{-11} \text{ Nm}^2\text{kg}^{-2}$

Mass of earth = $M = 6 \times 10^{24} \text{ kg}$

Required

Height of the satellite = $h = ?$

Solution

As we know that

$$g_h = \frac{GM_e}{(R+h)^2}$$

$$\frac{g}{4} = \frac{GM_e}{(R+h)^2}$$

Again we know that

$$M_e = \frac{gR^2}{G}$$

$$\text{or } GM_e = gR^2$$

Putting the value of GM_e in eq-----(1)

$$\frac{g}{4} = \frac{gR^2}{(R+h)^2}$$

$$\text{or } \frac{1}{4} = \frac{R^2}{(R+h)^2}$$

$$\text{or } (R+h)^2 = 4R^2$$

Taking square root on both sides

$$\sqrt{(R+h)^2} = \sqrt{4R^2}$$

$$R+h = 2R$$

$$h = 2R - R$$

$$h = R$$

Result

Required altitude is equal to one earth's radius.

5.9 A polar satellite is launched at 850 km above Earth. Find its orbital speed. (LHR 2014)

Given data:

Height of satellite = $h = 850 \text{ km}$

$$= 850 \times 1000$$

$$= 8.5 \times 10^5 \text{ m}$$

Mass of earth = $M_e = 6 \times 10^{24} \text{ kg}$

Gravitational constant = $G = 6.673 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$

Required:

Orbital speed of satellite = $V_o = ?$

Solution:

We know that

$$V_o \sqrt{g_h (R + h)}$$

Putting the value of g_h

$$V_o = \sqrt{\frac{GM_e}{(R + h)^2}} (R + h)$$

$$V_o = \sqrt{\frac{GM_e}{R + h}}$$

$$= \sqrt{\frac{(6.673 \times 10^{-11})(6 \times 10^{24})}{6.4 \times 10^6 + 8.5 \times 10^5}}$$

$$= \sqrt{\frac{4.0038 \times 10^{14}}{7250000}}$$

$$V_o = \sqrt{55224827.59}$$

$$V_o = 7431 \text{ ms}^{-1}$$

Result:

Orbital speed of satellite = $V_o = 7431 \text{ ms}^{-1}$

5.10 A communication satellite is launched at 42000 km above Earth. Find its orbital speed.

Given Data

Height of satellite = $h = 42000 \text{ km} = 42000 \times 10^3 \text{ m} = 4.2 \times 10^7 \text{ m}$

Mass of earth = $M = 6 \times 10^{24} \text{ kg}$

Gravitational constant = $G = 6.673 \times 10^{-11} \text{ Nm}^2\text{kg}^{-2}$

Required

Orbital speed of satellite = $V_o = ?$

Solution

As we know that

$$v_o = \sqrt{g_h (R + h)}$$

Putting the value of g_h

$$V_o = \sqrt{\frac{GM_e}{(R + h)^2}} (R + h)$$

$$V_o = \sqrt{\frac{GM_e}{R + h}}$$

By putting the values, we have

$$v_o = \sqrt{\frac{(6.673 \times 10^{-11})(6 \times 10^{24})}{6.4 \times 10^6 + 4.2 \times 10^7}}$$

$$v_o = \sqrt{\frac{4.0038 \times 10^{14}}{48400000}}$$

$$v_o = \sqrt{8272314.05}$$

$$v_o = 2876 \text{ ms}^{-1}$$

Result

Orbital speed of satellite = $v_o = 2876 \text{ ms}^{-1}$