

LINE BISECTORS AND ANGLE BISECTORS

Right Bisector of a Line Segment:

A line ℓ is called a right bisector of a line segment if ℓ is perpendicular to the line segment and passes through its mid-point.

Bisector of an Angle:

A ray BP is called the bisector of $\angle ABC$ if P is a point in the interior of the angle and $\angle ABP = \angle PBC$.

Theorem:

Any point on the right bisector of a line segment is equidistant from its end points.

Given:

A line LM intersects the line segment AB at the point C such that $\overline{LM} \perp \overline{AB}$ and $\overline{AC} \cong \overline{BC}$. P is a point on $\overline{LM}$.

To Prove: $\overline{PA} \cong \overline{PB}$

Construction:

Join P to the points A and B.

Proof

Statements	Reasons
In $\triangle ACP \leftrightarrow \triangle BCP$	
$\overline{AC} \cong \overline{BC}$	Given
$\angle ACP \cong \angle BCP$	given $\overline{PC} \perp \overline{AB}$, so that each $\angle$ at C = 90° .
$\overline{PC} \cong \overline{PC}$	common
$\therefore \triangle ACP \cong \triangle BCP$	S.A.S. postulate
Hence $\overline{PA} \cong \overline{PB}$	(corresponding sides of congruent triangles)

Theorem

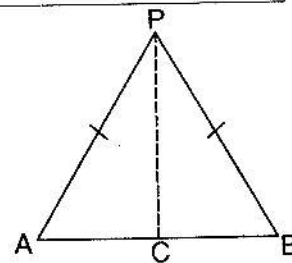
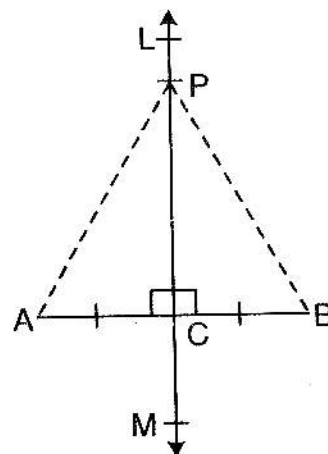
Any point equidistant from the end points of a line segment is on the right bisector of it.

Given

$\overline{AB}$ is a line segment. Point P is such that $\overline{PA} \cong \overline{PB}$.

To Prove

The Point P is on the right bisector of $\overline{AB}$.



Construction:

Joint P to C, the midpoint of $\overline{AB}$.

Proof:

Statements		Reasons
In	$\triangle ACP \leftrightarrow \triangle BCP$	
	$\overline{PA} \cong \overline{PB}$	Given
	$\overline{PC} \cong \overline{PC}$	Common
	$\overline{AC} \cong \overline{BC}$	Construction
	$\triangle ACP \cong \triangle BCP$	S.S.S $\cong$ S.S.S
	$\angle ACP \cong \angle BCP$(i)	(corresponding angles of congruent triangles)
But	$m\angle ACP + m\angle BCP = 180^\circ$(ii)	Supplementary angles
$\therefore$	$m\angle ACP = m\angle BCP = 90^\circ$	From (i) and (ii)
i.e.,	$\overline{PC} \perp \overline{AB}$(iii)	$m\angle ACP = 90^\circ$ (proved)
Also	$\overline{CA} \cong \overline{CB}$(iv)	
$\therefore$	$\overline{PC}$ is a right bisector of $\overline{AB}$.	construction
i.e.,	the point P is on the right bisector of $\overline{AB}$.	from (iii) and (iv)

Exercise 12.1

1. Prove that the centre of a circle is on the right bisectors of each of its chords.

Given

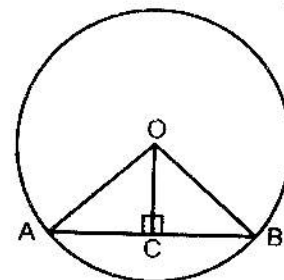
Circle with centre O

To Prove

Centre of the circle is on right bisectors of each of its chords

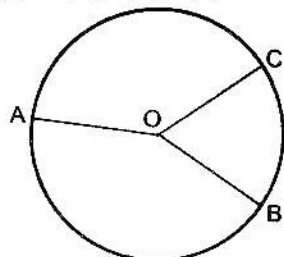
Construction

Draw any chord $\overline{AB}$. Draw $\overline{OC} \perp \overline{AB}$ join O with A and B.

Proof:

Statements		Reasons
In	$\triangle OAC \leftrightarrow \triangle OBC$	
	$\overline{OA} \cong \overline{OB}$	Radii of same circle
	$\overline{OC} \cong \overline{OC}$	Common
	$\angle ACO \cong \angle BCO$	Each of 90°
$\therefore$	$\triangle OAC \cong \triangle OBC$	H.S $\cong$ H.S
$\therefore$	$\overline{AC} \cong \overline{BC}$	Corresponding sides of the congruent triangles.
$\therefore$	$\overline{OC}$ is the right bisector of $\overline{AB}$	

2. Where will be the centre of a circle passing through three non-collinear points and why?

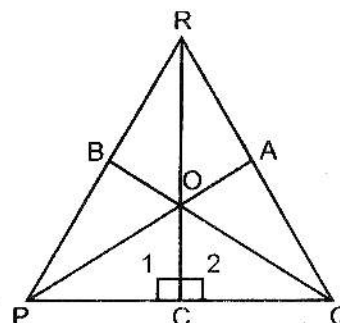


Circle is the locus of a point which moves so that its distance from a fixed point O remains same. Otherwise no circle will be formed.

3. Three villages P, Q and R are not on the same line. The people of these villages want to make a Children Park at such a place which is equidistant from these three villages. After fixing the place, of Children park, prove that the Park is equidistant from the three villages.

Proof:

Statements	Reasons
In $\triangle OPC \leftrightarrow \triangle OQC$	Construction
$\overline{CP} \cong \overline{CQ}$	Common
$\overline{OC} \cong \overline{OC}$	Each of 90°
$\angle 1 \cong \angle 2$	S.A.S $\cong$ S.A.S
$\therefore \triangle OPC \cong \triangle OQC$	Corresponding sides of congruent triangles
$\therefore \overline{OP} \cong \overline{OQ} \dots (i)$	
Similarly	
$\overline{OQ} \cong \overline{OR} \dots (ii)$	
$\therefore \overline{OP} \cong \overline{OQ} \cong \overline{OR}$	



Given

Three villages P, Q, R not on the same line.

To Prove

Park is equidistant from P, Q and R.

Construction

Complete the triangle PQR, draw the right bisectors of the sides $\overline{PQ}$ and $\overline{QR}$ cutting each other at O. Join O with P, Q and R. let O be the park.

Theorem.

The right bisectors of the sides of a triangle are concurrent.

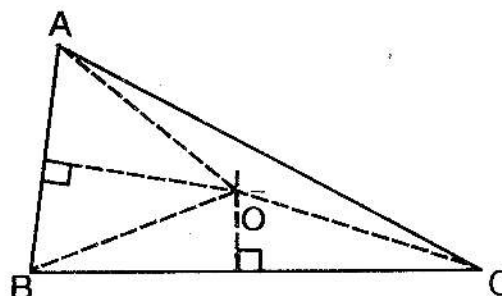
Given

$\triangle ABC$

To Prove

The right bisectors of $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ are concurrent.

Construction Draw the right bisectors of $\overline{AB}$ and $\overline{BC}$ which meet each other at the point O. Join O to A, B and C.

**Proof:**

Statements	Reasons
$\overline{OA} \cong \overline{OB}$(i)	(Each point on right bisector of a segment is equidistant from its end points)
$\overline{OB} \cong \overline{OC}$(ii)	as in (i)
$\overline{OA} \cong \overline{OC}$(iii)	From (i) and (ii)
$\therefore$ Point O is on the right bisector of $\overline{CA}$(iv)	(O is equidistant from A and C) construction
But point O is on the right bisector of $\overline{AB}$ and of $\overline{BC}$(v)	{from (iv) and (v)}
Hence the right bisectors of the three sides of a triangle are concurrent at O.	

Note:

- The right bisectors of the sides of an acute triangle intersect each other inside the triangle.
- The right bisectors of the sides of a right triangle intersect each other on the hypotenuse.
- The right bisectors of the sides of an obtuse triangle intersect each other outside the triangle.

Theorem

Any point on the bisector of an angle is equidistant from its arms.

Given

A point P is on $\overline{OM}$, the bisectors of $\angle AOB$.

To Prove

$\overline{PQ} \cong \overline{PR}$ i.e., P is equidistant from $\overline{OA}$ and $\overline{OB}$.

Construction

Draw $\overline{PR} \perp \overline{OA}$ and $\overline{PQ} \perp \overline{OB}$.

Proof:

Statements	Reasons
In $\triangle POQ \leftrightarrow \triangle POR$	
$\overline{OP} \cong \overline{OP}$	Common
$\angle PQO \cong \angle PRO$	Construction
$\angle POQ \cong \angle POR$	Given
$\therefore \triangle POQ \cong \triangle POR$	S.A.A. $\cong$ S.A.A.
Hence $\overline{PQ} \cong \overline{PR}$	(corresponding sides of congruent triangles)

Theorem

Any point inside an angle, equidistant from its arms, is on the bisector of it.

Given

Any point P lies inside $\angle AOB$ such that $\overline{PQ} \cong \overline{PR}$, where $\overline{PQ} \perp \overline{OB}$ and $\overline{PR} \perp \overline{OA}$.

To Prove

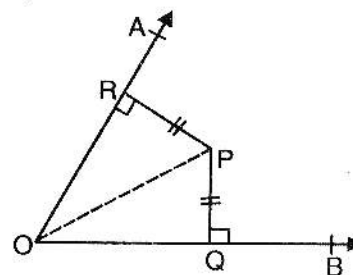
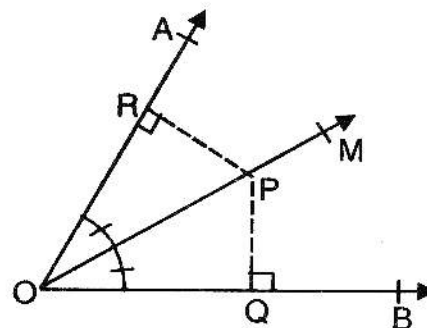
Point P is on the bisector of $\angle AOB$.

Construction

Join P to O.

Proof:

Statements	Reasons
In $\triangle POQ \leftrightarrow \triangle POR$	
$\angle PQO \cong \angle PRO$	Given (right angles)
$\overline{PO} \cong \overline{PO}$	Common
$\overline{PQ} \cong \overline{PR}$	Given
$\therefore \triangle POQ \cong \triangle POR$	H.S. $\cong$ H.S.
Hence $\angle POQ \cong \angle POR$	(corresponding angles of congruent triangles)
i.e., P is on the bisector of $\angle AOB$.	



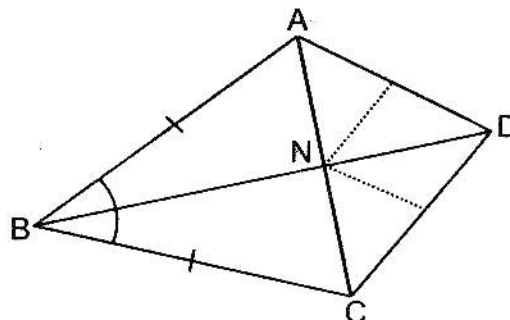
Exercise 12.2

1. In a quadrilateral $ABCD$, $\overline{AB} \cong \overline{BC}$ and the right bisectors of $\overline{AD}$, $\overline{CD}$ meet each other at point N . prove that $\overline{BN}$ is a bisector of $\angle ABC$.

Given Quadrilateral $ABCD$ in which $\overline{AB} \cong \overline{BC}$. Right bisectors of $\overline{AD}$ and $\overline{CD}$ meet each other at point N .

To prove $\overline{BN}$ is a bisector of $\angle ABC$

Construction Join N with A, B, C, D



Proof:

Statements	Reasons
$\overline{NC} \cong \overline{ND}$ (i)	N is on the right bisector of $\overline{CD}$
$\overline{NA} \cong \overline{ND}$ (ii)	N is on the right bisector of $\overline{AD}$
$\overline{NA} \cong \overline{NC}$ (iii)	By (i) and (ii)
In $\triangle ABN \leftrightarrow \triangle CBN$	
$\overline{AB} \cong \overline{BC}$	Given
$\overline{BN} \cong \overline{BN}$	Common
$\overline{NA} \cong \overline{NC}$	Proved
$\therefore \triangle ABN \cong \triangle CBN$	S.S.S $\cong$ S.S.S
$\angle ABN \cong \angle CBN$	Corresponding angles of congruent
$\therefore \overline{BN}$ is a bisector of $\angle ABC$.	triangles.

2. The bisectors of $\angle A$, $\angle B$ and $\angle C$ of a quadrilateral $ABCP$ meet each other at point O . Prove that the bisector of $\angle P$ will also pass through the point O .

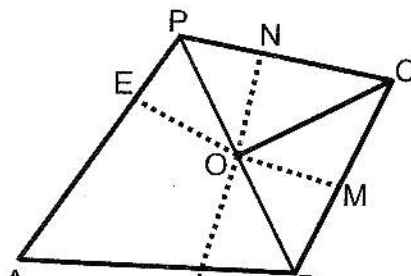
Given Bisector of the angles A, B, C meet at O .

To Prove

Bisector of $\angle P$ will also pass through O .

Construction

From O draw $\perp$ on the sides of quadrilateral BCP .

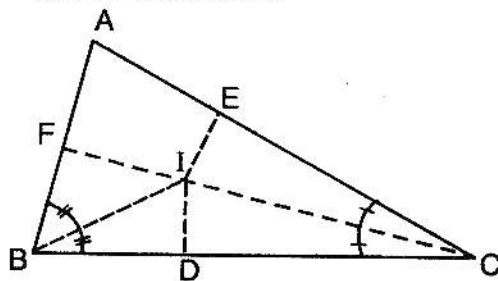


Proof:

Statements	Reasons
$\overline{OE} \cong \overline{OL}$ (i)	O is on the bisector of $\angle A$
$\overline{OL} \cong \overline{OM}$ (ii)	O is on the bisector of $\angle B$
$\overline{OM} \cong \overline{ON}$ (iii)	O is on the bisector of $\angle C$
$\therefore \overline{OE} \cong \overline{ON}$	By (i) and (ii), (iii)
$\therefore$ O is on the bisector of $\angle P$.	$\overline{OE} \cong \overline{ON}$

Theorem

The bisectors of the angles of a triangle are concurrent.



Given

$\triangle ABC$

To Prove

The bisectors of $\angle A$, $\angle B$ and $\angle C$ are concurrent.

Construction

Draw the bisectors of $\angle B$ and $\angle C$ which intersect at point I. From I, draw $\overline{IF} \perp \overline{AB}$, $\overline{ID} \perp \overline{BC}$ and $\overline{IE} \perp \overline{CA}$.

Proof:

Statements	Reasons
$\overline{ID} \cong \overline{IF}$ Similarly, $\overline{ID} \cong \overline{IE}$ $\therefore \overline{IE} \cong \overline{IF}$ So, the point I is on the bisector of $\angle A$ (i)	(Any point on bisector of an angle is equidistant from its arms) Each $\cong$ ID, proved.
Also the point I is on the bisectors of $\angle ABC$ and $\angle BCA$(ii)	Construction
Thus the bisectors of $\angle A$, $\angle B$ and $\angle C$ are concurrent at I.	{from (i) and (ii)}

Exercise

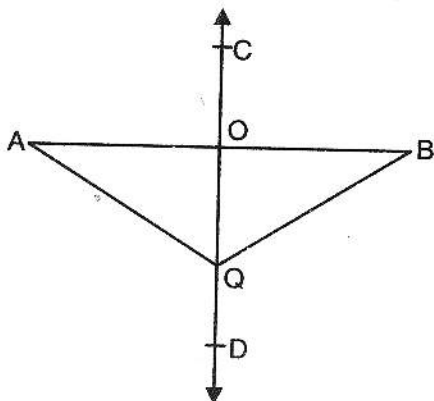
1. Which of the following are true and which are false?

- (i) Bisection means to divide into two equal parts. **(True)**
- (ii) Right bisection of line segment means to draw perpendicular which passes through the mid-point. **(True)**
- (iii) Any point on the right bisector of a line segment is not equidistant from its end points. **(False)**
- (iv) Any point equidistant from the end points of a line segment is on the right bisector of it. **(True)**
- (v) The right bisectors of the sides of a triangle are not concurrent. **(False)**
- (vi) The bisectors of the angles of a triangle are concurrent. **(True)**
- (vii) Any point on the bisector of an angle is not equidistant from its arms **(False)**
- (viii) Any point inside an angle, equidistant from its arms, is on the bisector of it. **(True)**

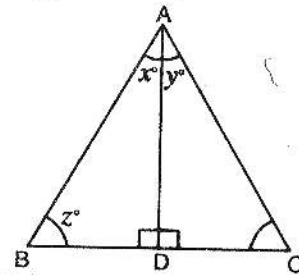
2. If $\overline{CD}$ is right bisector of line segment $\overline{AB}$, then:

(i) $m\overline{OA} = m\overline{OB}$

(ii) $m\overline{AQ} = m\overline{BQ}$



3. The given triangle ABC is equilateral triangle and $\overline{AD}$ is bisector of angle A, then find the values of unknowns x° , y° and z° .



$\therefore$ ABC is an equilateral triangle.

Its each angle = 60°

$\therefore z = 60^\circ$

$x + y = 60^\circ$

But $y = x$

$x + x = 60^\circ$

$2x = 60^\circ$

$x = \frac{60^\circ}{2}$

$x = 30^\circ$

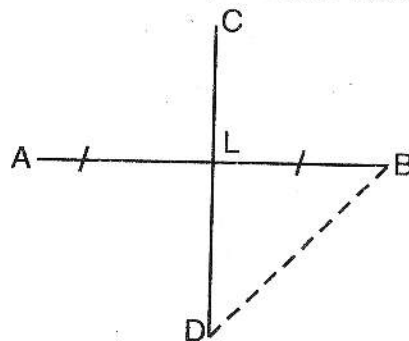
$\therefore y = 30^\circ$

Hence $z = 60^\circ$

4. $\overline{CD}$ is right bisector of the line segment $\overline{AB}$.

(i) if $m\overline{AB} = 6\text{cm}$, then find the $m\overline{AL}$ and $m\overline{LB}$.

(ii) If $m\overline{BD} = 4\text{cm}$, then find $m\overline{AD}$.



Given $\overline{CD}$ is a right bisector on the line segment $\overline{AB}$.

To find (i) $m\overline{AL}$, $m\overline{LB}$ when $m\overline{AB} = 6\text{cm}$

Proof:

(ii) $m\overline{AD}$ when $m\overline{BD} = 4\text{cm}$

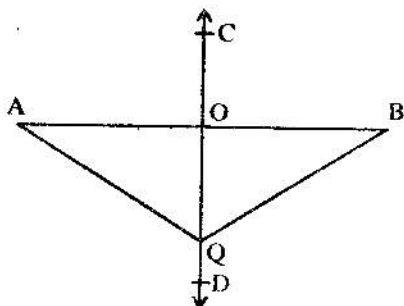
Construction Join B with D.

Statements	Reasons
(i) $m\overline{AL} = m\overline{LB}$ $m\overline{AL} = \frac{1}{2} m\overline{AB}$ $= \frac{1}{2} (6)$ $= 3\text{cm}$ $m\overline{LB} = m\overline{AL}$ $= 3\text{cm}.$	$\overline{CD}$ is a right bisector of $\overline{AB}$ $\therefore m\overline{AB} = 6\text{cm}$
(ii) $m\overline{AD} = m\overline{BD}$ $\therefore m\overline{AD} = 4\text{cm}$	$\therefore \overline{LD}$ is a right bisector of $\overline{AB}$ $\therefore m\overline{BD} = 4\text{cm}$

Objective

- Bisection means to divide into _____ equal parts
 (a) Two (b) Three
 (c) Four (d) Five
- _____ of line segment means to draw perpendicular which passes through the mid-point of line segment.
 (a) Right bisection (b) Bisection
 (c) Congruent (d) mid-point
- Any point on the _____ of a line segment is equidistant from its end points:
 (a) Right bisector (b) Angle bisector
 (c) Median (d) Altitude
- Any point equidistant from the end points of line segment is on the _____ of it:
 (a) Right bisector (b) Angle bisector
 (c) Median (d) Altitude
- The bisectors of the angles of a triangle are:
 (a) Concurrent (b) Congruent
 (c) Parallel (d) None
- Bisection of an angle means to draw a ray to divide the given angle into _____ equal parts:
 (a) Four (b) Three
 (c) Two (d) Five
- If $\overline{CD}$ is right bisector of line segment $\overline{AB}$ then: (i) $m\overline{OA} =$

- (a) $\overline{mOQ}$ (b) $\overline{mOB}$
 (c) $\overline{mAQ}$ (d) $\overline{mBQ}$



8. If $\overline{CD}$ is right bisector of line segment $\overline{AB}$, then $\overline{mAQ} = \underline{\hspace{1cm}}$
 (a) $\overline{mOA}$ (b) $\overline{mOB}$
 (c) $\overline{mBQ}$ (d) $\overline{mOD}$

9. The right bisectors of the sides of an acute triangle intersect each other the triangle.
 (a) Inside (b) Outside
 (c) Midpoint (d) None
10. The right bisectors of the sides of a right triangle intersect each other on the
 (a) Vertex (b) Midpoint
 (c) Hypotenuse (d) None
11. The right bisectors of the sides of an obtuse triangle intersect each other the triangle.
 (a) Outside (b) Inside
 (c) Midpoint (d) None

ANSWER KEY

1.	a	2.	a	3.	a	4.	a	5.	a
6.	c	7.	b	8.	c	9.	a	10.	c
11.	a								