Unit 01

QUADRATIC EQUATION

(1) Quadratic Equation:

An Equation, which contains the square of the unknown (variable) quantity, but not higher power, is called a quadratic equation or an equation of the second degree.

A second degree equation in one variable x of the form $ax^2+bx+c=0$, where $a\neq0$ and a, b, c are real numbers is called the general or standard form of a quadratic equation.

(2) Pure Quadratic Equation:

If b = 0 in quadratic equation $ax^2+bx+c=0$ then it is called a pure quadratic equation.

i.e $ax^2+0x+c=0$

$ax^2+c=0$

(3) Solution of Quadratic Equations:

To find solution set of a quadratic equation following methods are used:

(i) Factorization
(ii) Completing Square
(iii) Quadratic formula

Example 1:

Solve the quadratic equation $3x^2-6x = x + 20$ by Factorization.

Solution: $3x^2-6x = x + 20$

$3x^2 - 6x - x - 20 = 0$

$3x^2 - 7x - 20 = 0$

$3x^2 - 12x + 5x - 20 = 0$

$3x(x - 4) + 5(x - 4) = 0$

$(x - 4)(3x + 5) = 0$

Either $x - 4 = 0$ or $3x + 5 = 0$

$x = 4$ or $3x = -5$

or $x=\frac{-5}{3}$

Thus, the solution set is $\left\{\frac{-5}{3}, 4\right\}$

Example 2:

Solve $5x^2 = 30x$ by factorization

Solution: $5x^2 = 30x$

$5x^2 - 30x = 0$

Which is factorized as

$5x(x - 6) = 0$

Either $5x = 0$ or $x - 6 = 0$

$x=\frac{0}{5}$ or $x = 6$

$x = 0$ or $x = 6$

Thus the solution set is {0, 6}

Solution by Completing Square

Example 1: Solve the equation $x^2-3x-4 = 0$ by completing square.

Solution: $x^2 - 3x - 4 = 0$

Shifting constant term –4 to the right, we have

$x^2 - 3x = 4$

$(x)^2 - (x)(3) = 4$

$x^2-2(x)\left(\frac{3}{2}\right)=4$

Adding $\left(\frac{3}{2}\right)^2$ on the both sides we get

$(x)^2-2(x)\left(\frac{3}{2}\right)+\left(\frac{3}{2}\right)^2=4+\left(\frac{3}{2}\right)^2$

$\left(x-\frac{3}{2}\right)^2=4+\frac{9}{4}$

$\left(x-\frac{3}{2}\right)^2=\frac{16+9}{4}$

$$\left(x-\frac{3}{2}\right)^2=\frac{25}{4}$$

Taking square root of both sides, we get

$$\sqrt{\left(x-\frac{3}{2}\right)^2}=\pm\sqrt{\frac{25}{4}}$$

$$\left(x-\frac{3}{2}\right)=\pm\frac{5}{2}$$

$$x=\frac{3}{2}\pm\frac{5}{2}$$

$$x=\frac{3\pm5}{2}$$

$\Rightarrow$ $x=\frac{3+5}{2}$ or $x=\frac{3-5}{2}$

Either $x=\frac{8}{2}$ or $x=\frac{-2}{2}$

$\Rightarrow$ $x = 4$ or $x = -1$

$\therefore$ 4 and -1 are the roots of the given equation. Thus the solution set is $\{-1, 4\}$

Example 2:

Solve the equation $2x^2-5x-3=0$ by completing square.

Solution: $2x^2 - 5x - 3 = 0$

Dividing each term by 2

$$x^2-\frac{5}{2}x-\frac{3}{2}=0$$

Shifting constant term $\frac{-3}{2}$ to the right

$$x^2-\frac{5}{2}x=\frac{3}{2}$$

$$x^2-(x)\left(\frac{5}{2}\right)=\frac{3}{2}$$

$$x^2-\frac{2(x)}{2}\left(\frac{5}{2}\right)=\frac{3}{2}$$

$$(x)^2-2(x)\left(\frac{5}{4}\right)=\frac{3}{2}$$

Adding $\left(\frac{5}{4}\right)^2$ on both sides we get

$$(x)^2-2(x)\left(\frac{5}{4}\right)+\left(\frac{5}{4}\right)^2=\frac{3}{2}+\left(\frac{5}{4}\right)^2$$

$$\left(x-\frac{5}{4}\right)^2=\frac{3}{2}+\frac{25}{16}$$

$$\left(x-\frac{5}{4}\right)^2=\frac{24+25}{16}$$

$$\left(x-\frac{5}{4}\right)^2=\frac{49}{16}$$

Taking square root on both sides, we get

$$\sqrt{\left(x-\frac{5}{4}\right)^2}=\pm\sqrt{\frac{49}{16}}$$

$$x-\frac{5}{4}=\pm\frac{7}{4}$$

$$x=\frac{5}{4}\pm\frac{7}{4}$$

$$x=\frac{5\pm7}{4}$$

$\Rightarrow$ $x=\frac{5+7}{4}$ or $x=\frac{5-7}{4}$

$x=\frac{12}{4}$ or $x=\frac{-2}{4}$

$x = 3$ or $x=\frac{-1}{2}$

$\therefore$ $x=\frac{-1}{2}, 3$ are the roots of the given equation thus the solution set is $\left\{-\frac{1}{2}, 3\right\}$

EXERCISE 1.1

Q.1 Write the following quadratic equations in the Standard form and point out pure quadratic equations.

(i) $(x+7)(x-3)=-7$

Solution: $(x+7)(x-3)=-7$

$$x^2-3x+7x-21=-7$$
$$x^2+4x-21+7=0$$
$$x^2+4x-14=0$$

The standard form of quadratic equation is:

$$x^2+4x-14=0$$

(ii) $\dfrac{x^2+4}{3}-\dfrac{x}{7}=1$

Solution: $\dfrac{x^2+4}{3}-\dfrac{x}{7}=1$

$$\frac{7(x^2+4)-3x}{21}=1$$
$$7x^2+28-3x=21$$
$$7x^2-3x+28-21=$$
$$7x^2-3x+7=0$$

The standard form of quadratic equation is:

$$7x^2-3x+7=0$$

(iii) $\dfrac{x}{x+1}+\dfrac{x+1}{x}=6$

Solution: $\dfrac{x}{x+1}+\dfrac{x+1}{x}=6$

$$\frac{x^2+(x+1)^2}{x(x+1)}=6$$
$$x^2+x^2+1+2x=6x(x+1)$$
$$2x^2+2x+1=6x^2+6x$$
$$0=6x^2+6x-2x^2-2x-1$$
$$0=4x^2+4x-1$$
$$\Rightarrow \quad 4x^2+4x-1=0$$

The standard form of quadratic equation is:

$$4x^2+4x-1=0$$

(iv) $\left(\dfrac{x+4}{x-2}\right)-\left(\dfrac{x-2}{x}\right)+4=0$

Solution: $\left(\dfrac{x+4}{x-2}\right)-\left(\dfrac{x-2}{x}\right)+4=0$

$$\frac{(x+4)x-(x-2)^2+4x(x-2)}{(x-2)(x)}=0$$
$$x^2+4x-[x^2+2^2-2(x)(2)]+4x^2-8x=0$$
$$x^2+4x-x^2-4+4x+4x^2-8x=0$$
$$4x^2+4x+4x-8x-4=0$$
$$4x^2+8x-8x-4=0$$
$$4x^2-4=0$$
$$4(x^2-1)=0$$
$$\therefore \quad x^2-1=0 \quad (\because 4\neq 0)$$

So, $x^2-1=0$ is Pure Quadratic Equation

(v) $\dfrac{x+3}{x+4}-\dfrac{x-5}{x}=1$

Solution: $\dfrac{x+3}{x+4}-\dfrac{x-5}{x}=1$

$$\frac{x(x+3)-(x+4)(x-5)}{x(x+4)}=1$$
$$x^2+3x-(x^2-5x+4x-20)=1x(x+4)$$
$$x^2+3x-(x^2-1x-20)=x^2+4x$$
$$x^2+3x-x^2+x+20=x^2+4x$$
$$3x+x+20=x^2+4x$$
$$4x+20=x^2+4x$$
$$\Rightarrow \quad x^2+4x-4x-20=0$$
$$x^2-20=0$$
$$x^2+0x-20=0$$

As, $b=0$

So, $x^2-20=0$ is Pure Quadratic Equation

(vi) $\dfrac{x+1}{x+2}+\dfrac{x+2}{x+3}=\dfrac{25}{12}$

Solution: $\dfrac{x+1}{x+2}+\dfrac{x+2}{x+3}=\dfrac{25}{12}$

$$\frac{(x+1)(x+3)+(x+2)(x+2)}{(x+2)(x+3)}=\frac{25}{12}$$

$$\frac{(x^2+3x+1x+3)+(x^2+2x+2x+4)}{(x+2)(x+3)}=\frac{25}{12}$$

$$\frac{(x^2+4x+3)+(x^2+4x+4)}{x^2+3x+2x+6}=\frac{25}{12}$$

$$\frac{2x^2+8x+7}{x^2+5x+6}=\frac{25}{12}$$

$$12(2x^2+8x+7)=25(x^2+5x+6)$$

$$24x^2+96x+84=25x^2+125x+150$$

$$25x^2-24x^2+125x-96x+150-84=0$$

$$x^2+29x+66=0$$

The standard form of quadratic equation is:

$$x^2+29x+66=0$$

Q.2 Solve by Factorization:

(i) $\mathbf{x^2-x-20=0}$

Solution:

$$x^2-x-20=0$$

$$x^2-5x+4x-20=0$$

$$x(x-5)+4(x-5)=0$$

$$(x-5)(x+4)=0$$

$$x-5=0 \quad \text{or} \quad x+4=0$$

$$x=5 \quad \text{or} \quad x=-4$$

Solution set is $\{-4, 5\}$

(ii) $\mathbf{3y^2 = y(y-5)}$

Solution: $3y^2 = y(y-5)$

$$3y^2 = y^2-5y$$

$$3y^2-y^2+5y=0$$

$$2y^2+5y=0$$

$$y(2y+5)=0$$

Either $y=0$ or $2y+5=0$

$$y=0 \quad \text{or} \quad 2y=-5$$

$$y=0 \quad \text{or} \quad y=\frac{-5}{2}$$

Solution is $\left\{0, \dfrac{-5}{2}\right\}$

(iii) $\mathbf{4-32x=17x^2}$

Solution: $4-32x=17x^2$

$$17x^2+32x-4=0$$

$$17x^2+34x-2x-4=0$$

$$17x(x+2)-2(x+2)=0$$

$$(x+2)(17x-2)=0$$

Either $x+2=0$ or $17x-2=0$

$$x=-2 \quad \text{or} \quad 17x=2$$

$$x=-2 \quad \text{or} \quad x=\frac{2}{17}$$

Solution set is $\left\{-2, \dfrac{2}{17}\right\}$

(iv) $\mathbf{x^2-11x=152}$

Solution: $x^2-11x=152$

$$x^2-11x-152=0$$

$$x^2-19x+8x-152=0$$

$$x(x-19)+8(x-19)=0$$

$$(x-19)(x+8)=0$$

$$x-19=0 \quad \text{or} \quad x+8=0$$

$$x=19 \quad \text{or} \quad x=-8$$

Solution set is $\{-8, 19\}$

(v) $\dfrac{x+1}{x}+\dfrac{x}{x+1}=\dfrac{25}{12}$

Solution: $\dfrac{x+1}{x}+\dfrac{x}{x+1}=\dfrac{25}{12}$

$\dfrac{(x+1)^2+x^2}{x(x+1)}=\dfrac{25}{12}$

$\dfrac{(x)^2+(1)^2+2(x)(1)+x^2}{x^2+x}=\dfrac{25}{12}$

$\dfrac{x^2+1+2x+x^2}{x^2+x}=\dfrac{25}{12}$

$12(2x^2+2x+1)=25\,(x^2+x)$

$24x^2+24x+12=25x^2+25x$

$0=25x^2+25x-24x^2-24x-12$

$0=x^2+x-12$

$\Rightarrow \quad x^2+x-12=0$

$x^2+4x-3x-12=0$

$x(x+4)-3\,(x+4)=0$

$(x+4)(x-3)=0$

Either

$x+4=0 \quad$ or $\quad x-3=0$

$\Rightarrow \quad x=-4 \quad$ or $\quad x=3$

Solution set is $\{3,-4\}$

(vi) $\dfrac{2}{x-9}=\dfrac{1}{x-3}-\dfrac{1}{x-4}$

Solution: $\dfrac{2}{x-9}=\dfrac{1}{x-3}-\dfrac{1}{x-4}$

$\dfrac{2}{x-9}=\dfrac{(x-4)-(x-3)}{(x-3)(x-4)}$

$\dfrac{2}{x-9}=\dfrac{\cancel{x}-4-\cancel{x}+3}{x^2-4x-3x+12}$

$\dfrac{2}{x-9}=\dfrac{-1}{x^2-7x+12}$

$2(x^2-7x+12)=-1(x-9)$

$2x^2-14x+24=-1x+9$

$2x^2-14x+x+24-9=0$

$2x^2-13x+15=0$

$2x^2-10x-3x+15=0$

$2x(x-5)-3(x-5)=0$

$(x-5)(2x-3)=0$

Either $x-5=0 \quad$ or $\quad 2x-3=0$

$x=5 \quad$ or $\quad 2x=3$

$x=5 \quad$ or $\quad x=\dfrac{3}{2}$

Solution set is $\left\{5, \dfrac{3}{2}\right\}$

Q.3 Solve the following equations by completing square.

(i) $\mathbf{7x^2+2x-1=0}$

Solution: $7x^2+2x-1=0$

Dividing each term of the equation by 7

$\dfrac{\cancel{7}}{\cancel{7}}x^2+\dfrac{2}{7}x-\dfrac{1}{7}=0$

$x^2+\dfrac{2}{\cancel{2}}(x)\left(\dfrac{\cancel{2}}{7}\right)^1-\dfrac{1}{7}=0$

$(x)^2+2(x)\left(\dfrac{1}{7}\right)=\dfrac{1}{7}$

Adding $\left(\frac{1}{7}\right)^2$ on both sides

$$(x)^2+2(x)\left(\frac{1}{7}\right)+\left(\frac{1}{7}\right)^2=\frac{1}{7}+\left(\frac{1}{7}\right)^2$$

$$\left(x+\frac{1}{7}\right)^2=\frac{1}{7}+\frac{1}{49}$$

$$\left(x+\frac{1}{7}\right)^2=\frac{7+1}{49}$$

$$\left(x+\frac{1}{7}\right)^2=\frac{8}{49}$$

Taking square root of both sides

$$\sqrt{\left(x+\frac{1}{7}\right)^2}=\pm\sqrt{\frac{8}{49}}$$

$$\sqrt{\left(x+\frac{1}{7}\right)^2}=\pm\sqrt{\frac{4\times2}{49}}$$

$$x+\frac{1}{7}=\pm\frac{2\sqrt{2}}{7}$$

$$x=\frac{-1}{7}\pm\frac{2\sqrt{2}}{7}$$

$$\therefore \quad x=\frac{-1\pm2\sqrt{2}}{7}$$

Solution set is $\left\{\frac{-1\pm2\sqrt{2}}{7}\right\}$

(ii) $\mathbf{ax^2+4x-a=0}$, $a\neq0$

Solution: $ax^2+4x-a=0$, $a\neq0$

Dividing each term of the equation by 'a'

$$\frac{ax^2}{a}+4\frac{x}{a}-\frac{a}{a}=\frac{0}{a}$$

$$x^2+4\frac{x}{a}-1=0$$

$$x^2+\frac{2}{2}(x)\left(\frac{2}{a}\right)=1$$

$$(x)^2+2(x)\left(\frac{2}{a}\right)=1$$

Adding $\left(\frac{2}{a}\right)^2$ on both sides

$$(x)^2+2(x)\left(\frac{2}{a}\right)+\left(\frac{2}{a}\right)^2=1+\left(\frac{2}{a}\right)^2$$

$$\left(x+\frac{2}{a}\right)^2=1+\frac{4}{a^2}$$

$$\left(x+\frac{2}{a}\right)^2=\frac{a^2+4}{a^2}$$

Taking square root of both sides

$$\sqrt{\left(x+\frac{2}{a}\right)^2}=\pm\sqrt{\frac{a^2+4}{a^2}}$$

$$x+\frac{2}{a}=\pm\frac{\sqrt{a^2+4}}{a}$$

$$x=\frac{-2}{a}\pm\frac{\sqrt{a^2+4}}{a}$$

$$x=\frac{-2\pm\sqrt{a^2+4}}{a}$$

Solution set is $\left\{\frac{-2\pm\sqrt{a^2+4}}{a}\right\}$

(iii) $\mathbf{11x^2-34x+3=0}$

Solution: $11x^2-34x+3=0$

Dividing each term of the equation by 11

$$\frac{11}{11}x^2-\frac{34}{11}x+\frac{3}{11}=0$$

$$x^2-\frac{34}{11}x=-\frac{3}{11}$$

$$(x)^2-\frac{2}{2}(x)\left(\frac{17}{\frac{34}{11}}\right)=-\frac{3}{11}$$

$$(x)^2-2(x)\left(\frac{17}{11}\right)=-\frac{3}{11}$$

Adding $\left(\frac{17}{11}\right)^2$ on both sides we get

$$(x)^2 - 2(x)\left(\frac{17}{11}\right) + \left(\frac{17}{11}\right)^2 = -\frac{3}{11} + \left(\frac{17}{11}\right)^2$$

$$\left(x - \frac{17}{11}\right)^2 = \frac{-3}{11} + \frac{289}{121}$$

$$\left(x - \frac{17}{11}\right)^2 = \frac{-33 + 289}{121}$$

$$\left(x - \frac{17}{11}\right)^2 = \frac{256}{121}$$

Taking square root

$$\sqrt{\left(x - \frac{17}{11}\right)^2} = \pm\sqrt{\frac{256}{121}}$$

$$x - \frac{17}{11} = \pm\frac{16}{11}$$

$$x = \frac{17}{11} \pm \frac{16}{11}$$

$$x = \frac{17 \pm 16}{11}$$

$\Rightarrow$ $x = \frac{17-16}{11}$ or $x = \frac{17+16}{11}$

$x = \frac{1}{11}$ or $x = \frac{33}{11}$

$x = 3$

Solution set is $\left\{3, \frac{1}{11}\right\}$

(iv) $\ell x^2 + mx + n = 0$, $\ell \neq 0$

Solution: $\ell x^2 + mx + n = 0$, $\ell \neq 0$

Dividing each term of the equation by ℓ

$$\frac{\ell x^2}{\ell} + (x)\left(\frac{m}{\ell}\right)x = -\frac{n}{\ell}$$

$$x^2 + \frac{2}{2}(x)\left(\frac{m}{\ell}\right) = -\frac{n}{\ell}$$

$$x^2 + 2(x)\left(\frac{m}{2\ell}\right) = -\frac{n}{\ell}$$

Adding $\left(\frac{m}{2\ell}\right)^2$ on both sides

$$(x)^2 + 2(x)\left(\frac{m}{2\ell}\right) + \left(\frac{m}{2\ell}\right)^2 = -\frac{n}{\ell} + \left(\frac{m}{2\ell}\right)^2$$

$$\left(x + \frac{m}{2\ell}\right)^2 = -\frac{n}{\ell} + \frac{m^2}{4\ell^2}$$

$$\left(x + \frac{m}{2\ell}\right)^2 = \frac{-4\ell n + m^2}{4\ell^2}$$

Taking square root

$$\sqrt{\left(x + \frac{m}{2\ell}\right)^2} = \pm\sqrt{\frac{m^2 - 4\ell n}{4\ell^2}}$$

$$x + \frac{m}{2\ell} = \pm\frac{\sqrt{m^2 - 4\ell n}}{2\ell}$$

$$x = -\frac{m}{2\ell} \pm \frac{\sqrt{m^2 - 4\ell n}}{2\ell}$$

$$x = \frac{-m \pm \sqrt{m^2 - 4\ell n}}{2\ell}$$

Solution set is $\left\{\frac{-m \pm \sqrt{m^2 - 4\ell n}}{2\ell}\right\}$

(v) $3x^2 + 7x = 0$

Solution: $3x^2 + 7x = 0$

Dividing each term of equation by 3

$$\frac{\cancel{3}x^2}{\cancel{3}} + \frac{7}{3}x = 0$$

$$x^2 + \frac{2}{2}(x)\left(\frac{7}{3}\right) = 0$$

$$(x)^2 + 2(x)\left(\frac{7}{6}\right) = 0$$

Adding $\left(\frac{7}{6}\right)^2$ on both sides

$$(x)^2+2(x)\left(\frac{7}{6}\right)+\left(\frac{7}{6}\right)^2=\left(\frac{7}{6}\right)^2$$

$$\left(x+\frac{7}{6}\right)^2=\frac{49}{36}$$

Taking square root

$$\sqrt{\left(x+\frac{7}{6}\right)^2}=\pm\sqrt{\frac{49}{36}}$$

$$\left(x+\frac{7}{6}\right)=\pm\frac{7}{6}$$

$$x=\frac{-7}{6}\pm\frac{7}{6}$$

$$x=\frac{-7\pm7}{6}$$

$$\Rightarrow \quad x=\frac{-7-7}{6} \quad \text{or} \quad x=\frac{-7+7}{6}$$

$$x=\frac{-14}{6} \quad \text{or} \quad x=\frac{0}{6}$$

$$x=\frac{-7}{3} \quad \text{or} \quad x=0$$

Solution set is $\left\{0, \frac{-7}{3}\right\}$

(vi) $\mathbf{x^2-2x-195=0}$

Solution: $x^2-2x-195=0$

$$x^2-2x=195$$

$$x^2-\frac{2}{2}(x)(1)=195$$

$$(x)^2-2(x)(1)=195$$

Adding $(1)^2$ on both sides

$$(x)^2-2(x)(1)+(1)^2=195+(1)^2$$

$$(x-1)^2=195+1$$

$$(x-1)^2=196$$

Taking square root

$$\sqrt{(x-1)^2}=\pm\sqrt{196}$$

$$x-1=\pm14$$

$$x=1\pm14$$

$$\Rightarrow \quad x=1-14 \quad \text{or} \quad x=1+14$$

$$x=-13 \quad \text{or} \quad x=15$$

Solution set is $\{-13, 15\}$

(vii) $\mathbf{-x^2+\frac{15}{2}=\frac{7}{2}x}$

Solution: $-x^2+\frac{15}{2}=\frac{7}{2}x$

$$\frac{15}{2}=x^2+\frac{7}{2}x$$

$$\Rightarrow \quad x^2+\frac{7}{2}x=\frac{15}{2}$$

$$x^2+\frac{2}{2}(x)\left(\frac{7}{2}\right)=\frac{15}{2}$$

$$(x)^2+2(x)\left(\frac{7}{4}\right)=\frac{15}{2}$$

Adding $\left(\frac{7}{4}\right)^2$ on both sides

$$(x)^2+2(x)\left(\frac{7}{4}\right)+\left(\frac{7}{4}\right)^2=\frac{15}{2}+\left(\frac{7}{4}\right)^2$$

$$\left(x+\frac{7}{4}\right)^2=\frac{15}{2}+\frac{49}{16}$$

$$\left(x+\frac{7}{4}\right)^2=\frac{120+49}{16}$$

$$\left(x+\frac{7}{4}\right)^2=\frac{169}{16}$$

Taking Square Root

$$\sqrt{\left(x+\frac{7}{4}\right)^2}=\pm\sqrt{\frac{169}{16}}$$

$$x+\frac{7}{4}=\pm\frac{13}{4}$$

$$x=\frac{-7}{4}\pm\frac{13}{4}$$

EXERCISE 1.1

Q.1 Write the following quadratic equations in the Standard form and point out pure quadratic equations.

(i) $(x+7)(x-3)=-7$

Solution: $(x+7)(x-3)=-7$

$$x^2-3x+7x-21=-7$$
$$x^2+4x-21+7=0$$
$$x^2+4x-14=0$$

The standard form of quadratic equation is:

$$x^2+4x-14=0$$

(ii) $\dfrac{x^2+4}{3}-\dfrac{x}{7}=1$

Solution: $\dfrac{x^2+4}{3}-\dfrac{x}{7}=1$

$$\frac{7(x^2+4)-3x}{21}=1$$
$$7x^2+28-3x=21$$
$$7x^2-3x+28-21=0$$
$$7x^2-3x+7=0$$

The standard form of quadratic equation is:

$$7x^2-3x+7=0$$

(iii) $\dfrac{x}{x+1}+\dfrac{x+1}{x}=6$

Solution: $\dfrac{x}{x+1}+\dfrac{x+1}{x}=6$

$$\frac{x^2+(x+1)^2}{x(x+1)}=6$$
$$x^2+x^2+1+2x=6x(x+1)$$
$$2x^2+2x+1=6x^2+6x$$
$$0=6x^2+6x-2x^2-2x-1$$
$$0=4x^2+4x-1$$
$$\Rightarrow \quad 4x^2+4x-1=0$$

The standard form of quadratic equation is:

$$4x^2+4x-1=0$$

(iv) $\left(\dfrac{x+4}{x-2}\right)-\left(\dfrac{x-2}{x}\right)+4=0$

Solution: $\left(\dfrac{x+4}{x-2}\right)-\left(\dfrac{x-2}{x}\right)+4=0$

$$\frac{(x+4)x-(x-2)^2+4x(x-2)}{(x-2)(x)}=0$$
$$x^2+4x-[x^2+2^2-2(x)(2)]+4x^2-8x=0$$
$$x^2+4x-x^2-4+4x+4x^2-8x=0$$
$$4x^2+4x+4x-8x-4=0$$
$$4x^2+8x-8x-4=0$$
$$4x^2-4=0$$
$$4(x^2-1)=0$$
$$\therefore \quad x^2-1=0 \quad (\because 4\neq 0)$$

So, $x^2-1=0$ is Pure Quadratic Equation

(v) $\dfrac{x+3}{x+4}-\dfrac{x-5}{x}=1$

Solution: $\dfrac{x+3}{x+4}-\dfrac{x-5}{x}=1$

$$\frac{x(x+3)-(x+4)(x-5)}{x(x+4)}=1$$
$$x^2+3x-(x^2-5x+4x-20)=1x(x+4)$$
$$x^2+3x-(x^2-1x-20)=x^2+4x$$
$$x^2+3x-x^2+x+20=x^2+4x$$
$$3x+x+20=x^2+4x$$
$$4x+20=x^2+4x$$
$$\Rightarrow \quad x^2+4x-4x-20=0$$
$$x^2-20=0$$
$$x^2+0x-20=0$$

As, $b=0$

So, $x^2-20=0$ is Pure Quadratic Equation

(x) $7(x+2a)^2+3a^2=5a(7x+23a)$

Solution: $7(x+2a)^2+3a^2=5a(7x+23a)$

$7(x^2+4a^2+4ax)+3a^2=35ax+115a^2$

$7x^2+28a^2+28ax+3a^2-35ax-115a^2=0$

$7x^2-35ax+28ax+28a^2+3a^2-115a^2=0$

$7x^2-7ax-84a^2=0$

Dividing each term of the equation by 7 we get

$$\frac{7x^2}{7}-\frac{7ax}{7}=\frac{^{12}84a^2}{7}$$

$$x^2-ax=12a^2$$

$$(x)^2-2(x)\left(\frac{a}{2}\right)=12a^2$$

Adding $\left(\frac{a}{2}\right)^2$ an both sides

$$(x)^2-2(x)\left(\frac{a}{2}\right)+\left(\frac{a}{2}\right)^2=12a^2+\left(\frac{a}{2}\right)^2$$

$$\left(x-\frac{a}{2}\right)^2=12a^2+\frac{a^2}{4}$$

$$\left(x-\frac{a}{2}\right)^2=\frac{48a^2+a^2}{4}$$

$$\left(x-\frac{a}{2}\right)^2=\frac{49a^2}{4}$$

Taking square root

$$\sqrt{\left(x-\frac{a}{2}\right)^2}=\pm\sqrt{\frac{49a^2}{4}}$$

$$x-\frac{a}{2}=\pm\frac{7a}{2}$$

$$x=\frac{a}{2}\pm\frac{7a}{2}$$

$$x=\frac{a\pm7a}{2}$$

$\Rightarrow$ $x=\frac{a+7a}{2}$ or $x=\frac{a-7a}{2}$

$x=\frac{8a}{2}$ or $x=\frac{-6a}{2}$

$x=4a$ or $x=-3a$

Solution set is $\{-3a, 4a\}$

Quadratic Formula

Derivation of Quadratic Formula By Using Completing Square Method

The quadratic equation in standard form is

$$ax^2+bx+c=0, \quad a\neq 0$$

Dividing each term of the equation by 'a' we get.

$$\frac{ax^2}{a}+\frac{b}{a}x+\frac{c}{a}=0$$

Shifting constant term $\frac{c}{a}$ to the right, we have

$$x^2+\frac{b}{a}x=\frac{-c}{a}$$

$$x^2+\frac{2}{2}(x)\left(\frac{b}{a}\right)=-\frac{c}{a}$$

$$(x)^2+2(x)\left(\frac{b}{2a}\right)=\frac{-c}{a}$$

Adding $\left(\frac{b}{2a}\right)^2$ on both sides we obtain

$$(x)^2+2(x)\left(\frac{b}{2a}\right)+\left(\frac{b}{2a}\right)^2=\left(\frac{b}{2a}\right)^2-\frac{c}{a}$$

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2}{4a^2}-\frac{c}{a}$$

or $\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}$

Taking square root of the both sides, we get

$$\sqrt{\left(x+\frac{b}{2a}\right)^2}=\pm\sqrt{\frac{b^2-4ac}{4a^2}}$$

$\Rightarrow$ $x+\frac{b}{2a}=\pm\frac{\sqrt{b^2-4ac}}{2a}$

$\Rightarrow$ $x=-\frac{b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$

or $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Thus, $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$, $a\neq 0$ is known as "quadratic Formula"

Example 1: **Solve the Quadratic equation $2 + 9x = 5x^2$ by using quadratic formula.**

Solution: $2 + 9x = 5x^2$

The given equation in standard form can be written as

$$5x^2 - 9x - 2 = 0$$

Comparing with the standard quadratic equation $ax^2 + bx + c = 0$, we observe that

$$a = 5, \quad b = -9, \quad c = -2$$

Putting the values of a , b and c in quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \quad \text{we have}$$

$$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(5)(-2)}}{2(5)}$$

Or $$x = \frac{9 \pm \sqrt{81+40}}{10} = \frac{9 \pm \sqrt{121}}{10} = \frac{9 \pm 11}{10}$$

Either $x = \frac{9+11}{10}$ or $x = \frac{9-11}{10}$

$$x = \frac{20}{10} = 2 \quad \text{or} \quad x = \frac{-2}{10} = -\frac{1}{5}$$

$2, \frac{-1}{5}$ are the roots of the given equation

Thus, the solution set is $\left\{\frac{-1}{5}, 2\right\}$

Example 2: **Using quadratic formula, solve the equation** $\frac{2x+1}{x+2} - \frac{x-2}{x+4} = 0$

Solution: $\frac{2x+1}{x+2} - \frac{x-2}{x+4} = 0$

Simplifying and writing in the standard form

$$\frac{(2x+1)(x+4) - (x-2)(x+2)}{(x+2)(x+4)} = 0$$

$$(2x + 1)(x + 4) - (x - 2)(x + 2) = 0$$

$$2x^2 + 8x + x + 4 - (x^2 - 4) = 0$$

$$2x^2 + 9x + 4 - x^2 + 4 = 0$$

Or $$x^2 + 9x + 8 = 0$$

$$ax^2 + bx + c = 0$$

Here $a = 1, \quad b = 9, \quad c = 8$

Using quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \text{ we have}$$

$$x = \frac{-9 \pm \sqrt{(9)^2 - 4 \times 1 \times 8}}{2 \times 1}$$

$$= \frac{-9 \pm \sqrt{81 - 32}}{2} = \frac{-9 \pm \sqrt{49}}{2} = \frac{-9 \pm 7}{2}$$

$$x = \frac{-9+7}{2} \qquad x = \frac{-9-7}{2}$$

$$x = \frac{-2}{2} = -1 \qquad x = \frac{-16}{2} = -8$$

$\therefore$ $-1, -8$, are the roots of the given equation. Thus, the solution set is $\{-8, -1\}$

EXERCISE 1.2

Q.1 Solve the following equations using quadratic formula:

(i) $2 - x^2 = 7x$

Solution: $2 - x^2 = 7x$

$1x^2 + 7x - 2 = 0$

As $ax^2 + bx + c = 0$

$\Rightarrow$ $a = 1,\ b = 7,\ c = -2$

Using Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(7) \pm \sqrt{(7)^2 - 4(1)(-2)}}{2(1)}$$

$$x = \frac{-7 \pm \sqrt{49 + 8}}{2}$$

$\therefore$ $$x = \frac{-7 \pm \sqrt{57}}{2}$$

Solution set is $\left\{\frac{-7 \pm \sqrt{57}}{2}\right\}$

(ii) $5x^2 + 8x + 1 = 0$

Solution: $5x^2 + 8x + 1 = 0$

As $ax^2 + bx + c = 0$

$\Rightarrow$ $a = 5$, $b = 8$, $c = 1$

Using formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-8 \pm \sqrt{(8)^2 - 4(5)(1)}}{2(5)}$$

$$x = \frac{-8 \pm \sqrt{64 - 20}}{10}$$

$$x = \frac{-8 \pm \sqrt{44}}{10}$$

$$x = \frac{-8 \pm \sqrt{4 \times 11}}{10}$$

$$x = \frac{-8 \pm 2\sqrt{11}}{10}$$

$$x = \frac{\cancel{2}\left(-4 \pm \sqrt{11}\right)}{\cancel{10}\ 5}$$

$$x = \frac{-4 \pm \sqrt{11}}{5}$$

Solution set is $\left\{\frac{-4 \pm \sqrt{11}}{5}\right\}$

(iii) $\sqrt{3}x^2 + x = 4\sqrt{3}$

Solution: $\sqrt{3}x^2 + x = 4\sqrt{3}$

$\sqrt{3}\ x^2 + 1x - 4\sqrt{3} = 0$

As $ax^2 + bx + c = 0$

$\Rightarrow$ $a = \sqrt{3}$, $b = 1$, $c = -4\sqrt{3}$

Using formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4\left(\sqrt{3}\right)\left(-4\sqrt{3}\right)}}{2\sqrt{3}}$$

$$x = \frac{-1 \pm \sqrt{1 + 16\left(\sqrt{3}\right)^2}}{2\sqrt{3}}$$

$$x = \frac{-1 \pm \sqrt{1 + 16(3)}}{2\sqrt{3}}$$

$$x = \frac{-1 \pm \sqrt{1 + 48}}{2\sqrt{3}}$$

$$x = \frac{-1 \pm \sqrt{49}}{2\sqrt{3}}$$

$$x = \frac{-1 \pm 7}{2\sqrt{3}}$$

$\Rightarrow$ $$x = \frac{-1 - 7}{2\sqrt{3}} \quad \text{or} \quad x = \frac{-1 + 7}{2\sqrt{3}}$$

$$x = \frac{-\cancel{8}^{4}}{\cancel{2}\sqrt{3}} \quad \text{or} \quad x = \frac{\cancel{6}^{3}}{\cancel{2}\sqrt{3}}$$

$$x = \frac{-4}{\sqrt{3}} \quad \text{or} \quad x = \frac{3}{\sqrt{3}}$$

$$x = \frac{-4}{\sqrt{3}} \quad \text{or} \quad x = \sqrt{3} \qquad \left(\because \frac{a}{\sqrt{a}} = \sqrt{a}\right)$$

Solution set is $\left\{\sqrt{3}, -\frac{4}{\sqrt{3}}\right\}$

(iv) $\mathbf{4x^2 - 14 = 3}$

Solution: $4x^2 - 14 = 3x$

$$4x^2 - 3x - 14 = 0$$

As $ax^2 + bx + c = 0$

$\Rightarrow$ $a = 4, b = -3, c = -14$

Using formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(4)(-14)}}{2(4)}$$

$$x = \frac{3 \pm \sqrt{9 + 224}}{8}$$

Solution set is $\left\{\frac{3 \pm \sqrt{233}}{8}\right\}$

(v) $\mathbf{6x^2 - 3 - 7x = 0}$

Solution: $\mathbf{6x^2 - 3 - 7x = 0}$

$$6x^2 - 7x - 3 = 0$$

As $ax^2 + bx + c = 0$

$\Rightarrow$ $a = 6$, $b = -7$, $c = -3$

Using Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(6)(-3)}}{2(6)}$$

$$x = \frac{7 \pm \sqrt{49 + 72}}{12}$$

$$x = \frac{7 \pm \sqrt{121}}{12}$$

$$x = \frac{7 \pm 11}{12}$$

$$\Rightarrow \quad x = \frac{7 - 11}{12} \quad \text{or} \quad x = \frac{7 + 11}{12}$$

$$x = \frac{-4}{12} \quad \text{or} \quad x = \frac{18}{12}$$

$$x = \frac{-1}{3} \quad \text{or} \quad x = \frac{3}{2}$$

Solution set is $\left\{\frac{-1}{3}, \frac{3}{2}\right\}$

(vi) $\mathbf{3x^2 + 8x + 2 = 0}$

Solution: $3x^2 + 8x + 2 = 0$

As $ax^2 + bx + c = 0$

$\Rightarrow$ $a = 3$, $b = 8$, $c = 2$

Using formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-8 \pm \sqrt{(8)^2 - 4(3)(2)}}{2(3)}$$

$$x = \frac{-8 \pm \sqrt{64 - 24}}{6}$$

$$x = \frac{-8 \pm \sqrt{40}}{6}$$

$$x = \frac{-8 \pm \sqrt{4 \times 10}}{6}$$

$$x = \frac{-8 \pm 2\sqrt{10}}{6}$$

$$x = \frac{\cancel{2}\left(-4 \pm \sqrt{10}\right)}{\cancel{6}3}$$

$$x = \frac{-4 \pm \sqrt{10}}{3}$$

Solution set is $\left\{\frac{-4 \pm \sqrt{10}}{3}\right\}$

(vii) $\dfrac{3}{x-6}-\dfrac{4}{x-5}=1$

Solution: $\dfrac{3}{x-6}-\dfrac{4}{x-5}=1$

$$\frac{3(x-5)-4(x-6)}{(x-6)(x-5)}=1$$

$$3(x-5)-4(x-6)=(x-6)(x-5)$$

$$3x-15-4x+24=x^2-5x-6x+30$$

$$-1x+9=x^2-11x+30$$

$$x^2-11x+1x+30-9=0$$

$$1x^2-10x+21=0$$

As $ax^2+bx+c=0$

$\Rightarrow$ $a=1\ ,\ b=-10\ ,\ c=21$

Using formula:

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-10)\pm\sqrt{(-10)^2-4(1)(21)}}{2(1)}$$

$$x=\frac{10\pm\sqrt{100-84}}{2}$$

$$x=\frac{10\pm\sqrt{16}}{2}$$

$$x=\frac{10\pm4}{2}$$

$$x=\frac{\cancel{2}(5\pm2)}{\cancel{2}}$$

$\Rightarrow$ $x=5+2$ or $x=5-2$

$x=7$ or $x=3$

Solution set is $\{3, 7\}$

(viii) $\dfrac{x+2}{x-1}-\dfrac{4-x}{2x}=2\dfrac{1}{3}$

Solution: $\dfrac{x+2}{x-1}-\dfrac{4-x}{2x}=2\dfrac{1}{3}$

$$\frac{(x+2)2x-(4-x)(x-1)}{(x-1)(2x)}=\frac{7}{3}$$

$$\frac{(2x^2+4x)-(4x-4-x^2+x)}{2x^2-2x}=\frac{7}{3}$$

$$\frac{2x^2+4x-5x+4+x^2}{2x^2-2x}=\frac{7}{3}$$

$$\frac{3x^2-x+4}{2x^2-2x}=\frac{7}{3}$$

$$3(3x^2-x+4)=7(2x^2-2x)$$

$$9x^2-3x+12=14x^2-14x$$

$\Rightarrow$ $14x^2-9x^2-14x+3x-12=0$

$$5x^2-11x-12=0$$

As $ax^2+bx+c=0$

$\Rightarrow$ $a=5\ ,\ b=-11\ ,\ c=-12$

Using formula:

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-11)\pm\sqrt{(-11)^2-4(5)(-12)}}{2(5)}$$

$$x=\frac{11\pm\sqrt{121+240}}{10}$$

$$x=\frac{11\pm\sqrt{361}}{10}$$

$$x=\frac{11\pm19}{10}$$

$\Rightarrow$ $x=\dfrac{11-19}{10}$ or $x=\dfrac{11+19}{10}$

$x=\dfrac{-8}{10}$ or $x=\dfrac{30}{10}$

$x=-\dfrac{4}{5}$ or $x=3$

Solution set is $\left\{3, \dfrac{-4}{5}\right\}$

(ix) $\dfrac{a}{x-b}+\dfrac{b}{x-a}=2$

Solution: $\dfrac{a}{x-b}+\dfrac{b}{x-a}=2$

$$\frac{a(x-a)+b(x-b)}{(x-b)(x-a)}=2$$

$$\frac{ax-a^2+bx-b^2}{x^2-ax-bx+ab}=2$$

$$ax-a^2+bx-b^2=2(x^2-ax-bx+ab)$$

$$ax-a^2+bx-b^2=2x^2-2ax-2bx+2ab$$

$$2x^2-2ax-2bx+2ab-ax+a^2-bx+b^2=0$$

$$2x^2-2ax-ax-2bx-bx+2ab+a^2+b^2=0$$

$$2x^2-3ax-3bx+2ab+a^2+b^2=0$$

$$2x^2-(3a+3b)x+(a^2+b^2+2ab)=0$$

$$2x^2-(3a+3b)x+(a+b)^2=0$$

As $Ax^2+Bx+C=0$

$\Rightarrow$ $A=2\ ,\ B=-(3a+3b)\ ,\ C=(a+b)^2$

Using Formula

$$x=\frac{-B\pm\sqrt{B^2-4AC}}{2A}$$

$$x=\frac{-\left[-(3a+3b)\right]\pm\sqrt{\left[-(3a+3b)\right]^2-4(2)(a+b)^2}}{2(2)}$$

$$x=\frac{+3(a+b)\pm\sqrt{\left[-3(a+b)\right]^2-8(a+b)^2}}{4}$$

$$x=\frac{3(a+b)\pm\sqrt{\left[(-3)^2(a+b)\right]^2-8(a+b)^2}}{4}$$

$$x=\frac{3(a+b)\pm\sqrt{9(a+b)^2-8(a+b)^2}}{4}$$

$$x=\frac{3(a+b)\pm\sqrt{(a+b)^2}}{4}$$

$$x=\frac{3(a+b)\pm(a+b)}{4}$$

$$x=\frac{3(a+b)-(a+b)}{4} \quad \text{or} \quad x=\frac{3(a+b)+(a+b)}{4}$$

$$x=\frac{2(a+b)}{4} \quad \text{or} \quad x=\frac{4(a+b)}{4}$$

$$x=\frac{a+b}{2} \quad \text{or} \quad x=a+b$$

$$x=\frac{1}{2}(a+b) \quad \text{or} \quad x=a+b$$

Solution set is $\left\{(a+b),\ \frac{1}{2}(a+b)\right\}$

(x) $-(\ell+m)-\ell x^2+(2\ell+m)x=0\ ,\ \ell\neq 0$

Solution: $-(\ell+m)-\ell x^2+(2\ell+m)x=0\ ,\ \ell\neq 0$

$\Rightarrow$ $(\ell+m)+\ell x^2-(2\ell+m)x=0$ 01(032)

$\Rightarrow$ $\ell x^2-(2\ell+m)x+(\ell+m)=0$

As $ax^2+bx+c=0$

$\Rightarrow$ $a=\ell\ ,\ b=-(2\ell+m)\ ,\ c=\ell+m$

Using Formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-\left[-(2\ell+m)\right]\pm\sqrt{\left[-(2\ell+m)\right]^2-4(\ell)(\ell+m)}}{2(\ell)}$$

$$x=\frac{(2\ell+m)\pm\sqrt{(-1)^2(2\ell+m)^2-4\ell(\ell+m)}}{2\ell}$$

$$x=\frac{(2\ell+m)\pm\sqrt{(2\ell)^2+(m)^2+2(2\ell)(m)-4\ell^2-4\ell m}}{2\ell}$$

$$x=\frac{(2\ell+m)\pm\sqrt{\cancel{4\ell^2}+m^2+\cancel{4\ell m}-\cancel{4\ell^2}-\cancel{4\ell m}}}{2\ell}$$

$$x=\frac{(2\ell+m)\pm\sqrt{m^2}}{2\ell}$$

$$x=\frac{(2\ell+m)\pm m}{2\ell}$$

$$x=\frac{2\ell+\cancel{m}-\cancel{m}}{2\ell} \quad \text{or} \quad x=\frac{2\ell+m+m}{2\ell}$$

$$x=\frac{\cancel{2\ell}}{\cancel{2\ell}} \quad \text{or} \quad x=\frac{2\ell+2m}{2\ell}$$

$$x=1 \quad \text{or} \quad x=\frac{\cancel{2}(\ell+m)}{\cancel{2}\ell}$$

$$x=\frac{\ell+m}{\ell}$$

Solution set is $\left\{1,\ \frac{\ell+m}{\ell}\right\}$

Equations reducible to quadratic form

Type (i):

The equations of the type $\mathbf{ax^4 + bx^2 + c = 0}$

Example 1:

Solve the equation $x^4 - 13x^2 + 36 = 0$

Solution: $x^4 - 13x^2 + 36 = 0$ ..... (i)

Let $x^2 = y$ ..... (ii)

Then $x^4 = y^2$

Equation (i) becomes

$$y^2 - 13y + 36 = 0$$

$$y^2 - 9y - 4y + 36 = 0$$

$$y(y-9) - 4(y-9) = 0$$

$$(y-9)(y-4) = 0$$

Either $y - 9 = 0$ or $y - 4 = 0$,

$y = 9$ or $y = 4$

From equation (ii) Put $y = x^2$

$x^2 = 9$ or $x^2 = 4$

$\sqrt{x^2} = \pm\sqrt{9}$ or $\sqrt{x^2} = \pm\sqrt{4}$

$\Rightarrow$ $x = \pm 3$ or $x = \pm 2$

$\therefore$ The solution set is $\{\pm 2, \pm 3\}$

Type (ii):

The equations of the type $\mathbf{a\,p(x) + \frac{b}{p(x)} = c}$

Example 2:

Solve the equation $2(2x-1) + \frac{3}{2x-1} = 5$

Solution:

$2(2x-1) + \frac{3}{2x-1} = 5$ .....(i)

Let $2x - 1 = y$ .....(ii)

Then the equation (i) becomes

$$2y + \frac{3}{y} = 5$$

or $2y^2 + 3 = 5y$

$$2y^2 - 5y + 3 = 0$$

Using quadratic formula

$$ay^2 + by + c = 0$$

$\Rightarrow$ $a = 2$, $b = -5$, $c = 3$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 2 \times 3}}{2 \times 2}$$

$$y = \frac{5 \pm \sqrt{25-24}}{4} = \frac{5 \pm \sqrt{1}}{5} = \frac{5 \pm 1}{4}$$

We have

$y = \frac{5+1}{4}$ or $y = \frac{5-1}{4}$

$y = \frac{6}{4} = \frac{3}{2}$ or $y = \frac{4}{4} = 1$

From equation (ii) putting $(y = 2x - 1)$

$2x - 1 = \frac{3}{2}$ or $2x - 1 = 1$

$2x = \frac{3}{2} + 1$ or $2x = 1 + 1 = 2$

$2x = \frac{5}{2}$ or $x = \frac{2}{2}$

$x = \frac{5}{4}$ or $x = 1$

Thus, the solution set is $\left\{1, \frac{5}{4}\right\}$

Type (iii): Reciprocal equations of the type:

$$a\left(x^2 + \frac{1}{x^2}\right) + b\left(x + \frac{1}{x}\right) + c = 0$$

or $ax^4 + bx^3 + cx^2 + bx + a = 0$

Reciprocal Equation

An equation is said to be a reciprocal equation, if it remains unchanged, when x is replaced by $\frac{1}{x}$.

Replacing x by $\frac{1}{x}$ in

$ax^4 - bx^3 + cx^2 - bx + a = 0$, we have

$$a\left(\frac{1}{x}\right)^4 - b\left(\frac{1}{x}\right)^3 + c\left(\frac{1}{x}\right)^2 - b\left(\frac{1}{x}\right) + a = 0$$

$$\left(\frac{a}{x^4}\right) - \left(\frac{b}{x^3}\right) + \left(\frac{c}{x^2}\right) - \left(\frac{b}{x}\right) + a = 0$$

Multiplying both side by x^4, we get

$a - bx + cx^2 - bx^3 + ax^4 = 0$.

By arranging these terms we get the same equation .

Thus $ax^4 - bx^3 + cx^2 - bx + a = 0$ is a reciprocal equation.

Example 3:

Solve the equation

$2x^4 - 5x^3 - 14x^2 - 5x + 2 = 0$

Solution:

$2x^4 - 5x^3 - 14x^2 - 5x + 2 = 0$

Dividing each term by x^2.

$$\frac{2x^4}{x^2} - \frac{5x^3}{x^2} - \frac{14x^2}{x^2} - \frac{5x}{x^2} + \frac{2}{x^2} = 0$$

$$2x^2 - 5x - 14 - \frac{5}{x} + \frac{2}{x^2} = 0$$

$$2\left(x^2 + \frac{1}{x^2}\right) - 5\left(x + \frac{1}{x}\right) - 14 = 0 \qquad \ldots\ldots \text{(i)}$$

Let $x + \frac{1}{x} = y$, $\qquad \ldots\ldots$ (ii)

Taking square both side

$$x^2 + \frac{1}{x^2} + 2(x)\left(\frac{1}{x}\right) = y^2$$

$$x^2 + \frac{1}{x^2} = y^2 - 2$$

So equation (i) becomes

$2(y^2 - 2) - 5y - 14 = 0$

$2y^2 - 4 - 5y - 14 = 0$

$2y^2 - 5y - 18 = 0$

$2y^2 - 9y + 4y - 18 = 0$

$y(2y - 9) + 2(2y - 9) = 0$

$\Rightarrow \qquad (2y - 9)(y + 2) = 0$

Either $2y - 9 = 0$ or $y + 2 = 0$

From equation (ii) Putting $y = x + \frac{1}{x}$, we have

$2\left(x + \frac{1}{x}\right) - 9 = 0$ or $x + \frac{1}{x} + 2 = 0$

$2x + \frac{2}{x} - 9 = 0$ or $x^2 + 1 + 2x = 0$

$\boxed{2x^2 - 9x + 2 = 0}$ or $\boxed{x^2 + 2x + 1 = 0}$

First we solve $2x^2 - 9x + 2 = 0$ by quadratic formula, we get

$a = 2 , b = -9 , c = 2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4 \times 2 \times 2}}{2 \times 2}$$

$$x = \frac{9 \pm \sqrt{81 - 16}}{4}$$

$$x = \frac{9 \pm \sqrt{65}}{4}$$

Now we solve $x^2 + 2x + 1 = 0$ by quadratic formula

$a = 1 , b = 2 , c = 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4 \times 1 \times 1}}{2 \times 1}$$

$$x = \frac{-2 \pm \sqrt{4 - 4}}{2} \quad \text{or} \quad x = \frac{-2 \pm 0}{2}$$

$x = -1, -1$

Thus, the solution set is $\left\{-1, \frac{9 \pm \sqrt{65}}{4}\right\}$

Type (iv): Exponential equations:

In exponential equations, variable occurs in exponent. For example $2^x = 8$

Example 4:

Solve the equation $5^{1+x} + 5^{1-x} = 26$

Solution: $5^{1+x} + 5^{1-x} = 26$

$5^1 . 5^x + 5^1 . 5^{-x} = 26$

or $5.5^x + \frac{5}{5^x} - 26 = 0$ .....(i)

Let $5^x = y$. .....(ii)

Then equation (i) becomes

$5y + \frac{5}{y} - 26 = 0$

$5y^2 + 5 - 26y = 0$

$5y^2 - 26y + 5 = 0$

$5y^2 - 25y - y + 5 = 0$

$5y(y-5) - 1(y-5) = 0$

$(y-5)(5y-1) = 0$

Either $y - 5 = 0$ or $5y - 1 = 0$,

$y = 5$ or $5y = 1$

or $y = \frac{1}{5} = 5^{-1}$

From equation (ii) Put $y = 5^x$

$5^x = 5^1$ or $5^x = 5^{-1}$

$\Rightarrow x = 1$ or $\Rightarrow x = -1$

$\therefore$ The solution set is $\{\pm 1\}$.

Type (v): The equations of the type:

$(x+a)(x+b)(x+c)(x+d) = k$,

where $a + b = c + d$

Example 5:

Solve the equation

$(x-1)(x+2)(x+8)(x+5) = 19$

Solution:

$(x-1)(x+2)(x+8)(x+5) = 19$

Or $[(x-1)(x+8)][(x+2)(x+5)] - 19 = 0$

$(\because -1 + 8 = 2 + 5)$

$(x^2 + 7x - 8)(x^2 + 7x + 10) - 19 = 0$ .... (i)

Let $x^2 + 7x = y$ .... (ii)

Then equation (i) becomes

$(y-8)(y+10) - 19 = 0$

$y^2 + 2y - 80 - 19 = 0$

$y^2 + 2y - 99 = 0$

$y^2 + 11y - 9y - 99 = 0$

$y(y+11) - 9(y+11) = 0$

$(y+11)(y-9) = 0$

Either $y + 11 = 0$ or $y - 9 = 0$

From equation (ii) Put $y = x^2 + 7x$

$\boxed{x^2 + 7x + 11 = 0}$ or $\boxed{x^2 + 7x - 9 = 0}$

First we solve $x^2 + 7x + 11 = 0$ by quadratic formula

$a = 1$, $b = 7$, $c = 11$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-7 \pm \sqrt{(7)^2 - 4(1)(11)}}{2(1)}$

Or $= \frac{-7 \pm \sqrt{49 - 44}}{2} = \frac{-7 \pm \sqrt{5}}{2}$

Now we solve $x^2 + 7x - 9 = 0$

$a = 1$, $b = 7$, $c = -9$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-7 \pm \sqrt{(7)^2 - 4(1)(-9)}}{2(1)}$

or $x = \frac{-7 \pm \sqrt{49 + 36}}{2} = \frac{-7 \pm \sqrt{85}}{2}$

$\therefore$ The solution set is $\left\{\frac{-7 \pm \sqrt{5}}{2}, \frac{-7 \pm \sqrt{85}}{2}\right\}$

EXERCISE 1.3

Q.1 $2x^4 - 11x^2 + 5 = 0$

Solution: $2x^4 - 11x^2 + 5 = 0$ ..... (i)

Let $x^2 = y$ ..... (ii)

$(x^2)^2 = y^2$

$x^4 = y^2$

Put $x^2 = y$ and $x^4 = y^2$ in equation (i)

$2x^4 - 11x^2 + 5 = 0$

$2y^2 - 11y + 5 = 0$

$2y^2 - 10y - y + 5 = 0$

$2y(y-5) - 1(y-5) = 0$

$(y-5)(2y-1) = 0$

$y - 5 = 0$ or $2y - 1 = 0$

$y = 0 + 5$ or $2y = 1$

$y = 5$ or $y = \frac{1}{2}$

From equation (ii) Put $y = x^2$

$x^2 = 5$ or $x^2 = \frac{1}{2}$

$\sqrt{x^2} = \pm\sqrt{5}$ or $\sqrt{x^2} = \pm\sqrt{\frac{1}{2}}$

$x = \pm\sqrt{5}$ or $x = \pm\frac{1}{\sqrt{2}}$

The Solution set is $\left\{\pm\frac{1}{\sqrt{2}}, \pm\sqrt{5}\right\}$

Q.2 $2x^4 = 9x^2 - 4$

Solution: $2x^4 = 9x^2 - 4$

$2x^4 - 9x^2 + 4 = 0$ .....(i)

Let $x^2 = y$ .....(ii)

$(x^2)^2 = y^2$

$x^4 = y^2$

Put $x^2 = y$ and $x^4 = y^2$ in equation (i)

$2y^2 - 9y + 4 = 0$

$2y^2 - 8y - 1y + 4 = 0$

$2y(y-4) - 1(y-4) = 0$

$(y-4)(2y-1) = 0$

$y - 4 = 0$ or $2y - 1 = 0$

$y = 4$ or $2y = 1$

$\Rightarrow$ $y = 4$ or $y = \frac{1}{2}$

From equation (ii) put $y = x^2$

$x^2 = 4$ or $x^2 = \frac{1}{2}$

$\sqrt{x^2} = \pm\sqrt{4}$ or $\sqrt{x^2} = \pm\sqrt{\frac{1}{2}}$

$x = \pm 2$ or $x = \pm\frac{1}{\sqrt{2}}$

The Solution set is $\left\{\pm\frac{1}{\sqrt{2}}, \pm 2\right\}$

Q.3 $5x^{\frac{1}{2}} = 7x^{\frac{1}{4}} - 2$

Solution: $5x^{1/2} = 7x^{1/4} - 2$ .....(i)

Let $x^{\frac{1}{4}} = y$ .....(ii)

Taking square

$\left(x^{\frac{1}{4}}\right)^2 = y^2$

$x^{\frac{1}{2}} = y^2$

Put $x^{\frac{1}{4}} = y$ and $x^{\frac{1}{2}} = y^2$ in equation (i)

$5x^{\frac{1}{2}} = 7x^{\frac{1}{4}} - 2$

$5y^2 = 7y - 2$

$5y^2 - 7y + 2 = 0$

$5y^2 - 5y - 2y + 2 = 0$

$5y(y-1) - 2(y-1) = 0$

$(y-1)(5y-2) = 0$

$y - 1 = 0$ or $5y - 2 = 0$

$y = 1$ or $5y = 2$

$y = 1$ or $y = \frac{2}{5}$

From equation (ii) put $y = x^{\frac{1}{4}}$

$x^{\frac{1}{4}} = 1$ or $x^{\frac{1}{4}} = \frac{2}{5}$

$\left(x^{\frac{1}{4}}\right)^4 = (1)^4$ or $\left(x^{\frac{1}{4}}\right)^4 = \left(\frac{2}{5}\right)^4$

$x = 1$ or $x = \frac{2^4}{5^4}$

$x = \frac{16}{625}$

Solution set is $\left\{\frac{16}{625}, 1\right\}$

Q.4 $x^{\frac{2}{3}} + 54 = 15x^{\frac{1}{3}}$

Solution: $x^{\frac{2}{3}} + 54 = 15x^{\frac{1}{3}}$ .....(i)

Let $x^{\frac{1}{3}} = y$ .....(ii)

Taking square

$\left(x^{\frac{1}{3}}\right)^2 = y^2$

$x^{\frac{2}{3}} = y^2$

Put $x^{\frac{1}{3}} = y$ and $x^{\frac{2}{3}} = y^2$ in equation (i)

$x^{\frac{2}{3}} + 54 = 15x^{\frac{1}{3}}$

$y^2 + 54 = 15y$

$y^2 - 15y + 54 = 0$

$y^2 - 9y - 6y + 54 = 0$

$y(y-9) - 6(y-9) = 0$

$(y-9)(y-6) = 0$

$y - 9 = 0$ or $y - 6 = 0$

$y = 9$ or $y = 6$

From equation (ii) put $y = x^{\frac{1}{3}}$

$x^{\frac{1}{3}} = 9$ or $x^{\frac{1}{3}} = 6$

$\left(x^{\frac{1}{3}}\right)^3 = (9)^3$ or $\left(x^{\frac{1}{3}}\right)^3 = (6)^3$

$x = 9^3$ or $x = 6^3$

$x = 729$ or $x = 216$

Solution set is $\{216, 729\}$

Q.5 $3x^{-2} + 5 = 8x^{-1}$

Solution: $3x^{-2} + 5 = 8x^{-1}$ .....(i)

Let $x^{-1} = y$ .....(ii)

$(x^{-1})^2 = y^2$

$x^{-2} = y^2$

Put $x^{-1} = y$ and $x^{-2} = y^2$ in equation (i)

$3x^{-2} + 5 = 8x^{-1}$

$3y^2 + 5 = 8y$

$3y^2 - 8y + 5 = 0$

$3y^2 - 5y - 3y + 5 = 0$

$y(3y-5) - 1(3y-5) = 0$

$(3y-5)(y-1) = 0$

$3y - 5 = 0$ or $y - 1 = 0$

$3y = 5$ or $y = 1$

$y = \frac{5}{3}$

From equation (ii) put $y = x^{-1}$

$x^{-1} = \frac{5}{3}$ or $x^{-1} = 1$

$\frac{1}{x} = \frac{5}{3}$ or $\frac{1}{x} = 1$

$x = \frac{3}{5}$ or $x = 1$

Solution set is $\left\{\frac{3}{5}, 1\right\}$

Q.6 $(2x^2+1)+\dfrac{3}{2x^2+1}=4$

Solution: $(2x^2+1)+\dfrac{3}{2x^2+1}=4$ .....(i)

Let $2x^2+1=y$ .....(ii)

Replacing $2x^2+1$ by "y" in equation (i), we get

$$y+\frac{3}{y}=4$$

Multiplying both sides by "y"

$$y^2+3=4y$$
$$y^2-4y+3=0$$
$$y^2-3y-1y+3=0$$
$$y(y-3)-1(y-3)=0$$
$$(y-3)(y-1)=0$$
$$y-3=0 \quad \text{or} \quad y-1=0$$
$$y=3 \quad \text{or} \quad y=1$$

From equation (ii) Put $y = 2x^2+1$

$$2x^2+1=3 \quad \text{or} \quad 2x^2+1=1$$
$$2x^2=3-1 \quad \text{or} \quad 2x^2=1-1$$
$$2x^2=2 \quad \text{or} \quad 2x^2=0$$
$$x^2=\frac{2}{2} \quad \text{or} \quad x^2=\frac{0}{2}$$
$$x^2=1 \quad \text{or} \quad x^2=0$$
$$\sqrt{x^2}=\pm\sqrt{1} \quad \text{or} \quad \sqrt{x^2}=\pm\sqrt{0}$$
$$x=\pm1 \quad \text{or} \quad x=0$$

Solution set is $\{-1, 0, 1\}$

Q.7 $\dfrac{x}{x-3}+4\left(\dfrac{x-3}{x}\right)=4$

Solution: $\dfrac{x}{x-3}+4\left(\dfrac{x-3}{x}\right)=4$ .....(i)

Let $\dfrac{x}{x-3}=y$ .....(ii)

$$\frac{x-3}{x}=\frac{1}{y}$$

Equation (i) becomes

$$y+4\left(\frac{1}{y}\right)=4$$

Multiplying both sides by "y"

$$y^2+4=4y$$
$$y^2-4y+4=0$$
$$y^2-2y-2y+4=0$$
$$y(y-2)-2(y-2)=0$$
$$(y-2)(y-2)=0$$
$$(y-2)^2=0$$
$$y-2=0$$
$$y=2$$

From equation (ii) Put $y=\dfrac{x}{x-3}$

$$\frac{x}{x-3}=2$$
$$x=2(x-3)$$
$$x=2x-6$$
$$6=2x-x$$
$$6=x \Rightarrow x=6$$

Solution set is $\{6\}$

Q.8 $\dfrac{4x+1}{4x-1}+\dfrac{4x-1}{4x+1}=2\dfrac{1}{6}$

Solution: $\dfrac{4x+1}{4x-1}+\dfrac{4x-1}{4x+1}=2\dfrac{1}{6}$ ..... (i)

Let $\dfrac{4x+1}{4x-1}=y$ ..... (ii)

$$\frac{4x-1}{4x+1}=\frac{1}{y}$$

Equation (i) becomes

$$y+\frac{1}{y}=2\frac{1}{6}$$
$$y+\frac{1}{y}=\frac{13}{6}$$

Multiplying both sides by "6y"

$$6y^2+6=13y$$
$$6y^2-13y+6=0$$

$6y^2-9y-4y+6=0$

$3y(2y-3)-2(2y-3)=0$

$(2y-3)(3y-2)=0$

$2y-3=0$ or $3y-2=0$

$2y=3$ or $3y=2$

$y=\frac{3}{2}$ or $y=\frac{2}{3}$

From equation (ii) Put $y=\frac{4x+1}{4x-1}$

$\frac{4x+1}{4x-1}=\frac{3}{2}$ or $\frac{4x+1}{4x-1}=\frac{2}{3}$

$2(4x+1)=3(4x-1)$ or $3(4x+1)=2(4x-1)$

$8x+2=12x-3$ or $12x+3=8x-2$

$2+3=12x-8x$ or $12x-8x=-2-3$

$4x=5$ or $4x=-5$

$x=\frac{5}{4}$ or $x=\frac{-5}{4}$

Solution set is $\left\{\pm\frac{5}{4}\right\}$

Q.9 $\frac{x-a}{x+a}-\frac{x+a}{x-a}=\frac{7}{12}$

Solution: $\frac{x-a}{x+a}-\frac{x+a}{x-a}=\frac{7}{12}$ .....(i)

Let $\frac{x-a}{x+a}=y$ .....(ii)

Or $\frac{x+a}{x-a}=\frac{1}{y}$

The equation (i) becomes

$y-\frac{1}{y}=\frac{7}{12}$

$\frac{y^2-1}{y}=\frac{7}{12}$

$12(y^2-1)=7y$

$12y^2-12=7y$

$12y^2-7y-12=0$

$12y^2-16y+9y-12=0$

$4y(3y-4)+3(3y-4)=0$

$(3y-4)(4y+3)=0$

$3y-4=0$ or $4y+3=0$

$3y=4$ or $4y=-3$

$y=\frac{4}{3}$ or $y=\frac{-3}{4}$

From equation (ii) Putting $y=\frac{x-a}{x+a}$, we get

$\frac{x-a}{x+a}=\frac{4}{3}$ or $\frac{x-a}{x+a}=\frac{-3}{4}$

$3(x-a)=4(x+a)$ or $4(x-a)=-3(x+a)$

$3x-3a=4x+4a$ or $4x-4a=-3x-3a$

$-3a-4a=4x-3x$ or $4x+3x=-3a+4a$

$-7a=x$ or $7x=a$

$\Rightarrow$ $x=-7a$ or $x=\frac{a}{7}$

The solution set is $\left\{-7a, \frac{a}{7}\right\}$

Q.10 $x^4-2x^3-2x^2+2x+1=0$

Solution: $x^4-2x^3-2x^2+2x+1=0$

Dividing both sides by "x^2"

$\frac{x^4}{x^2}-\frac{2x^3}{x^2}-\frac{2x^2}{x^2}+\frac{2x}{x^2}+\frac{1}{x^2}=\frac{0}{x^2}$

$x^2-2x-2+\frac{2}{x}+\frac{1}{x^2}=0$

$\left(x^2+\frac{1}{x^2}\right)-2x+\frac{2}{x}-2=0$

$\left(x^2+\frac{1}{x^2}\right)-2\left(x-\frac{1}{x}\right)-2=0$ ....(i)

Let $x-\frac{1}{x}=y$ .....(ii)

Taking square of both sides

$\left(x-\frac{1}{x}\right)^2=y^2$

$x^2+\frac{1}{x^2}-2=y^2$

$x^2+\frac{1}{x^2}=y^2+2$

Putting these values in equation (i)

$$y^2 + 2^2 - 2(y) - 2^2 = 0$$

$$y^2 - 2y = 0$$

$$y(y-2) = 0$$

$$y = 0 \quad \text{or} \quad y - 2 = 0$$

$$y = 0 \quad \text{or} \quad y = 2$$

From equation (ii) Put $y = x - \frac{1}{x}$

$$x - \frac{1}{x} = 0 \quad \text{or} \quad x - \frac{1}{x} = 2$$

$$\frac{x^2 - 1}{x} = 0 \quad \text{or} \quad \frac{x^2 - 1}{x} = 2$$

$$x^2 - 1 = 0 \quad \text{or} \quad x^2 - 1 = 2x$$

$$x^2 = 1 \quad \text{or} \quad x^2 - 2x - 1 = 0$$

$$\sqrt{x^2} = \pm\sqrt{1} \quad \text{or} \quad \boxed{1x^2 - 2x - 1 = 0}$$

$$\boxed{x = \pm 1}$$

Now Solving $1x^2 - 2x - 1 = 0$ by quadratic formula

$$a = 1 \ , \ b = -2 \ , \ c = -1$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{+2 \pm \sqrt{4+4}}{2}$$

$$x = \frac{2 \pm \sqrt{8}}{2}$$

$$x = \frac{2 \pm \sqrt{4 \times 2}}{2}$$

$$x = \frac{2 \pm 2\sqrt{2}}{2}$$

$$x = \frac{\cancel{2}\left(1 \pm \sqrt{2}\right)}{\cancel{2}}$$

$$\boxed{x = 1 \pm \sqrt{2}}$$

The solution set is $\{\pm 1, \ 1 \pm \sqrt{2}\}$

Q.11 $2x^4 + x^3 - 6x^2 + x + 2 = 0$

Solution: $2x^4 + x^3 - 6x^2 + x + 2 = 0$

Dividing each terms by x^2

$$\frac{2x^4}{x^2} + \frac{x^3}{x^2} - \frac{6\cancel{x}^2}{\cancel{x}^2} + \frac{x}{x^2} + \frac{2}{x^2} = \frac{0}{x^2}$$

$$2x^2 + x - 6 + \frac{1}{x} + \frac{2}{x^2} = 0$$

$$2x^2 + \frac{2}{x^2} + x + \frac{1}{x} - 6 = 0$$

$$2\left(x^2 + \frac{1}{x^2}\right) + \left(x + \frac{1}{x}\right) - 6 = 0 \quad \ldots\ldots(i)$$

Let $x + \frac{1}{x} = y$ $\ldots\ldots(ii)$

Taking square of both sides

$$\left(x + \frac{1}{x}\right)^2 = y^2$$

$$x^2 + \frac{1}{x^2} + 2(\cancel{x})\left(\frac{1}{\cancel{x}}\right) = y^2$$

$$x^2 + \frac{1}{x^2} + 2 = y^2$$

$$x^2 + \frac{1}{x^2} = y^2 - 2$$

Put $x + \frac{1}{x} = y$ and $x^2 + \frac{1}{x^2} = y^2 - 2$ in equation (i)

$$2\left(x^2 + \frac{1}{x^2}\right) + \left(x + \frac{1}{x}\right) - 6 = 0$$

$$2(y^2 - 2) + y - 6 = 0$$

$$2y^2 - 4 + y - 6 = 0$$

$$2y^2 + y - 10 = 0$$

$$2y^2 + 5y - 4y - 10 = 0$$

$$y(2y+5) - 2(2y+5) = 0$$

$$(2y+5)(y-2) = 0$$

$$2y + 5 = 0 \quad \text{or} \quad y - 2 = 0$$

$$2y = -5 \quad \text{or} \quad y = 2$$

$$y = \frac{-5}{2}$$

From equation (ii) Put $y = x + \frac{1}{x}$

$x + \frac{1}{x} = \frac{-5}{2}$

$\frac{x^2+1}{x} = \frac{-5}{2}$

$2(x^2+1) = -5x$

$2x^2 + 2 = -5x$

$2x^2 + 5x + 2 = 0$

$2x^2 + 4x + 1x + 2 = 0$

$2x(x+2) + 1(x+2) = 0$

$(x+2)(2x+1) = 0$

$x + 2 = 0$ or $2x + 1 = 0$

$x = -2$ or $2x = -1$

$\boxed{x = -2}$ or $\boxed{x = \frac{-1}{2}}$

$x + \frac{1}{x} = 2$

$\frac{x^2+1}{x} = 2$

$x^2 + 1 = 2x$

$x^2 - 2x + 1 = 0$

$x^2 - 1x - 1x + 1 = 0$

$x(x-1) - 1(x-1) = 0$

$(x-1)(x-1) = 0$

$(x-1)^2 = 0$

$\sqrt{(x-1)^2} = \sqrt{0}$

$x - 1 = 0$

$x = 1$

Solution set is $\left\{1, -2, \frac{-1}{2}\right\}$

Q.12 $\mathbf{4.2^{2x+1} - 9.2^x + 1 = 0}$

Solution: $4.2^{2x+1} - 9.2^x + 1 = 0$

$4.2^{2x}2^1 - 9.2^x + 1 = 0$

$8(2^x)^2 - 9(2^x) + 1 = 0$ ..... (i)

Let $2^x = y$ ..... (ii)

The equation (i) becomes

$8y^2 - 9y + 1 = 0$

$8y^2 - 8y - 1y + 1 = 0$

$8y(y-1) - 1(y-1) = 0$

$(y-1)(8y-1) = 0$

$y - 1 = 0$ or $8y - 1 = 0$

$y = 1$ or $8y = 1$

or $y = \frac{1}{8}$

From equation (ii) Put $y = 2^x$

$2^x = 1$ or $2^x = \frac{1}{8}$

$2^x = 2^0$ or $2^x = \frac{1}{2^3}$

$\Rightarrow$ $x = 0$ or $2^x = 2^{-3}$

$\Rightarrow$ $x = -3$

Solution set is $\{-3, 0\}$

Q.13 $\mathbf{3^{2x+2} = 12.3^x - 3}$

Solution: $3^{2x+2} = 12.3^x - 3$

$3^{2x}\ 3^2 - 12.3^x + 3 = 0$

$9(3^x)^2 - 12.(3^x) + 3 = 0$ ..... (i)

Let $3^x = y$ ..... (ii)

Put $3^x = y$ in equation (i)

$9y^2 - 12y + 3 = 0$

$9y^2 - 9y - 3y + 3 = 0$

$9y(y-1) - 3(y-1) = 0$

$(y-1)(9y-3) = 0$

$y - 1 = 0$ or $9y - 3 = 0$

$y = 1$ or $9y = 3$

$\boxed{y = 1}$ or $y = \frac{3}{9} \Rightarrow \boxed{y = \frac{1}{3}}$

From equation (ii) Put $y = 3^x$

$3^x = 1$ or $3^x = \frac{1}{3}$

$3^x = 3^0$ or $3^x = 3^{-1}$

$\Rightarrow$ $x = 0$ or $x = -1$

The solution set is $\{0, -1\}$

Q.14 $\mathbf{2^x + 64.2^{-x} - 20 = 0}$

Solution: $2^x + 64.2^{-x} - 20 = 0$

$2^x + \frac{64}{2^x} - 20 = 0$ .....(i)

Let $2^x = y$ .....(ii)

Put $2^x = y$ in equation (i)

$y + \frac{64}{y} - 20 = 0$

Multiply both sides by "y"

$y^2 + 64 - 20y = 0$

$y^2 - 20y + 64 = 0$

$y^2 - 16y - 4y + 64 = 0$

$y(y-16) - 4(y-16) = 0$

$(y-16)\ (y-4) = 0$

$y - 16 = 0$ or $y - 4 = 0$

$y = 16$ or $y = 4$

From equation (ii) put $y = 2^x$

$2^x = 16$ or $2^x = 4$

$2^x = 2^4$ or $2^x = 2^2$

$\Rightarrow$ $x = 4$ or $x = 2$

The solution set is $\{2, 4\}$

Q.15 $(x+1)(x+3)(x-5)(x-7)=192$

Solution: $(x+1)(x+3)(x-5)(x-7)=192$

$\because \quad a+b=c+d$
$1-5=3-7$
$-4=-4$

$(x+1)(x-5)(x+3)(x-7)=192$

$(x^2-5x+1x-5)(x^2-7x+3x-21)=192$

$(x^2-4x-5)(x^2-4x-21)=192 \quad ....(i)$

Let $x^2-4x=y \quad .....(ii)$

The equation (i) becomes

$(y-5)(y-21)=192$

$y^2-21y-5y+105=192$

$y^2-26y+105-192=0$

$y^2-26y-87=0$

$y^2-29y+3y-87=0$

$y(y-29)+3(y-29)=0$

$(y-29)(y+3)=0$

$y-29=0 \quad \text{or} \quad y+3=0$

$y=29 \quad \text{or} \quad y=-3$

From equation (ii) Put $y=x^2-4x$

$x^2-4x=29 \quad \text{or} \quad x^2-4x=-3$

$x^2-4x-29=0 \quad \text{or} \quad x^2-4x+3=0$

$\boxed{x^2-4x-29=0} \quad \text{or} \quad \boxed{x^2-4x+3=0}$

First we solve $x^2-4x-29=0$ by quadratic formula

$a=1 \quad , \quad b=-4 \quad , \quad c=-29$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(-29)}}{2(1)}$$

$$x=\frac{4\pm\sqrt{16+116}}{2}$$

$$x=\frac{4\pm\sqrt{132}}{2}$$

$$x=\frac{4\pm\sqrt{4\times33}}{2}$$

$$x=\frac{4\pm2\sqrt{33}}{2}$$

$$x=\frac{\cancel{2}\left(2\pm\sqrt{33}\right)}{\cancel{2}}$$

$$\boxed{x=2\pm\sqrt{33}}$$

Now we solve $x^2-4x+3=0$ by factorization

$x^2-4x+3=0$

$x^2-3x-x+3=0$

$x(x-3)-1(x-3)=0$

$(x-3)(x-1)=0$

$x-3=0 \quad \text{or} \quad x-1=0$

$\boxed{x=3} \quad \text{or} \quad \boxed{x=1}$

Solution set is $\{1, 3, 2\pm\sqrt{33}\}$

Q.16 $(x-1)(x-2)(x-8)(x+5)+360=0$

Solution: $(x-1)(x-2)(x-8)(x+5)+360=0$

$\left[(x-1)(x-2)\right]\left[(x-8)(x+5)\right]+360=0$

$\left[x^2-2x-1x+2\right]\left[x^2+5x-8x-40\right]+360=0$

$(x^2-3x+2)(x^2-3x-40)+360=0 \quad ....(i)$

Let $x^2-3x=y \quad .....(ii)$

Put it in equation (i)

$(y+2)(y-40)+360=0$

$y^2-40y+2y-80+360=0$

$y^2-38y+280=0$

$y^2-28y-10y+280=0$

$y(y-28)-10(y-28)=0$

$(y-28)(y-10)=0$

$y-28=0 \quad \text{or} \quad y-10=0$

$y=28 \quad \text{or} \quad y=10$

From equation (ii) Put $y=x^2-3x$

$x^2-3x=28$	$x^2-3x=10$
$x^2-3x-28=0$	$x^2-3x-10=0$
$x^2-7x+4x-28=0$	$x^2-5x+2x-10=0$
$x(x-7)+4(x-7)=0$	$x(x-5)+2(x-5)=0$
$(x-7)(x+4)=0$	$(x-5)(x+2)=0$
$x-7=0$ or $x+4=0$	$x-5=0$ or $x+2=0$
$x=7$ or $x=-4$	$x=5$ or $x=-2$

The solution set is

$\{5, -2, 7, -4\}$

Radical Equation

An equation involving expression of the variable under radical sign is called radical equation. For example $\sqrt{x+1}=2$

(i) The equation of the type $\sqrt{ax+b}=cx+d$

Example 1:

Solve the equation $\sqrt{3x+7}=2x+3$

Solution: $\sqrt{3x+7}=2x+3$ ..... (i)

Squaring both sides of the equation (i), we get

$$\left(\sqrt{3x+7}\right)^2=(2x+3)^2$$

$$3x+7=(2x)^2+2(2x)(3)+(3)^2$$

$$3x+7=4x^2+12x+9$$

$$0=4x^2+12x+9-3x-7$$

$\Rightarrow$ $4x^2+9x+2=0$

Applying quadratic formula,

$a=4$, $b=9$, $c=2$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-9\pm\sqrt{(9)^2-4\times4\times2}}{2\times4}$$

$$x=\frac{-9\pm\sqrt{81-32}}{8}=\frac{-9\pm\sqrt{49}}{8}=\frac{-9\pm7}{8}$$

$$x=\frac{-9+7}{8} \quad \text{or} \quad x=\frac{-9-7}{8}$$

$$x=\frac{-2}{8} \quad \text{or} \quad x=\frac{-16}{8}$$

$$x=\frac{-1}{4} \quad \text{or} \quad x=-2$$

Checking: Putting $x=-\frac{1}{4}$ in the equation (i), we have

$$\sqrt{3\left(-\frac{1}{4}\right)+7}=2\left(-\frac{1}{4}\right)+3$$

$\Rightarrow$ $$\sqrt{\frac{-3+28}{4}}=-\frac{1}{2}+3$$

$\Rightarrow$ $$\sqrt{\frac{25}{4}}=\frac{-1+6}{2}$$

$$\frac{5}{2}=\frac{5}{2}$$ which is true.

Putting $x=-2$ in the equation (i), we get

$$\sqrt{3(-2)+7}=2(-2)+3$$

$\Rightarrow$ $\sqrt{1}=-1$

$1\neq-1$

On checking, we find that $x=-2$ does not satisfy the equation (i), so it is an extraneous root. Thus the solution set is $\left\{-\frac{1}{4}\right\}$

(ii) The equation of the type $\sqrt{x+a}+\sqrt{x+b}=\sqrt{x+c}$

Example 2: Solve the equation

$\sqrt{x+3}+\sqrt{x+6}=\sqrt{x+11}$

Solution: $\sqrt{x+3}+\sqrt{x+6}=\sqrt{x+11}$ ....(i)

Squaring both sides of the equation (i), we have

$$\left(\sqrt{x+3}+\sqrt{x+6}\right)^2=\left(\sqrt{x+11}\right)^2$$

$$\left(\sqrt{x+3}\right)^2+\left(\sqrt{x+6}\right)^2+2\left(\sqrt{x+3}\right)\left(\sqrt{x+6}\right)=x+11$$

$$x+3+x+6+2\left(\sqrt{x+3}\right)\left(\sqrt{x+6}\right)=x+11$$

$$2x+9+2\sqrt{(x+3)(x+6)}=x+11$$

$$2\sqrt{x^2+6x+3x+18}=x+11-2x-9$$

or $$2\sqrt{x^2+9x+18}=-x+2$$

Again taking square of both sides we get

$$(2)^2\left(\sqrt{x^2+9x+18}\right)^2=(-x)^2+(2)^2+2(-x)(2)$$

$$4(x^2+9x+18)=x^2+4-4x$$

$$(4x^2+36x+72)=x^2-4x+4$$

$$4x^2+36x+72-x^2+4x-4=0$$

$$3x^2+40x+68=0$$

Applying quadratic formula

$a=3$, $b=40$, $c=68$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-40\pm\sqrt{(40)^2-4(3)(68)}}{2(3)}$$

$$x=\frac{-40\pm\sqrt{1600-816}}{6}$$

$$x=\frac{-40\pm\sqrt{784}}{6}=\frac{-40\pm28}{6}$$

We have

$$x=\frac{-40+28}{6} \qquad x=\frac{-40-28}{6}$$

$$x=\frac{-12}{6} \qquad x=\frac{-68}{6}$$

$$x=-2 \qquad x=\frac{-34}{3}$$

Checking:

Putting $x=-2$ in (i)

$$\sqrt{-2+3}+\sqrt{-2+6}=\sqrt{-2+11}$$

$$\sqrt{1}+\sqrt{4}=\sqrt{9}$$

$$1+2=3$$

$3=3$ which is true

So one root is 3

Now Putting $x=\frac{-34}{3}$ in the equation (i), we have

$$\sqrt{\frac{-34}{3}+3}+\sqrt{\frac{-34}{3}+6}=\sqrt{\frac{-34}{3}+11}$$

or

$$\sqrt{\frac{-34+9}{3}}+\sqrt{\frac{-34+18}{3}}=\sqrt{\frac{-34+33}{3}}$$

$$\Rightarrow \quad \sqrt{\frac{25}{3}\times(-1)}+\sqrt{\frac{16}{3}\times(-1)}=\sqrt{\frac{1}{3}\times(-1)}$$

$$\Rightarrow \quad \frac{5}{\sqrt{3}}i+\frac{4}{\sqrt{3}}i=\frac{1}{\sqrt{3}}i \qquad \left(\because i=\sqrt{-1}\right)$$

$$\frac{9}{\sqrt{3}}i=\frac{1}{\sqrt{3}}i \text{ which is not true.}$$

As $\frac{-34}{3}$ is an extraneous root,

So, the solution set is $\{-2\}$.

(iii) The equation of the type $\sqrt{x^2+px+m}+\sqrt{x^2+px+n}=q$

Example 3: **Solve the equation**

$$\sqrt{x^2-3x+36}-\sqrt{x^2-3x+9}=3$$

Solution: $\sqrt{x^2-3x+36}-\sqrt{x^2-3x+9}=3$ ...(i)

Let $x^2-3x=y$ .....(ii)

Put it in equation (i)

Then $\sqrt{y+36}-\sqrt{y+9}=3$

Squaring both sides

$$\left(\sqrt{y+36}-\sqrt{y+9}\right)^2=(3)^2$$

$$\left(\sqrt{y+36}\right)^2+\left(\sqrt{y+9}\right)^2-2\left(\sqrt{y+36}\right)\left(\sqrt{y+9}\right)=9$$

$$y+36+y+9-2\sqrt{(y+36)(y+9)}=9$$

$$2y+45-2\sqrt{\left(y^2+36y+9y+324\right)}=9$$

$$-2\sqrt{\left(y^2+45y+324\right)}=9-2y-45$$

$$-2\sqrt{y^2+45y+324}=-2y-36$$

$$\cancel{-2}\sqrt{y^2+45y+324}=\cancel{-2}(y+18)$$

Again squaring both sides

$$\left(\sqrt{y^2+45y+324}\right)^2=(y+18)^2$$

$$y^2+45y+324=(y)^2+2(y)(18)+(18)^2$$

$$\cancel{y^2}+45y+\cancel{324}=\cancel{y^2}+36y+\cancel{324}$$

$$45y-36y=0$$

$$9y=0$$

$\Rightarrow$ $y=0$

From equation (ii) put $y=x^2-3x$ so

$$x^2-3x=0$$

$\Rightarrow$ $x(x-3)=0$

Either $x=0$ or $x-3=0 \Rightarrow x=3$

$\therefore$ $x=0, 3$ are the roots of the equation.

Thus, the solution set is $\{0, 3\}$

EXERCISE 1.4

Solve the following equations:

Q.1 $2x+5=\sqrt{7x+16}$

Solution: $2x+5=\sqrt{7x+16}$ .....(i)

Squaring both sides

$$(2x+5)^2=\left(\sqrt{7x+16}\right)^2$$

$$(2x)^2+(5)^2+2(2x)(5)=7x+16$$

$$4x^2+25+20x=7x+16$$

$$4x^2+20x-7x+25-16=0$$

$$4x^2+13x+9=0$$

$$4x^2+4x+9x+9=0$$

$$4x(x+1)+9(x+1)=0$$

$$(x+1)(4x+9)=0$$

$x+1=0$ or $4x+9=0$

$x=-1$ or $4x=-9$

$x=-1$ or $x=\frac{-9}{4}$

Checking

Putting $x=-1$ in the equation (i) we have

$$2x+5=\sqrt{7x+16}$$

$$2(-1)+5=\sqrt{7(-1)+16}$$

$$-2+5=\sqrt{-7+16}$$

$$3=\sqrt{9}$$

$3=3$ which is true

Putting $x=\frac{-9}{4}$ in the Equation (i) we have

$$2x+5=\sqrt{7x+16}$$

$$2\left(\frac{-9}{4}\right)+5=\sqrt{7\left(\frac{-9}{4}\right)+16}$$

$$\frac{-18}{4}+5=\sqrt{\frac{-63}{4}+16}$$

$$\frac{-18+20}{4}=\sqrt{\frac{-63+64}{4}}$$

$$\frac{2}{4}=\sqrt{\frac{1}{4}} \quad \Rightarrow \frac{1}{2}=\frac{1}{2}$$ which is true

So, the solution set is $\left\{-1, \frac{-9}{4}\right\}$

Q.2 $\sqrt{x+3}=3x-1$

Solution: $\sqrt{x+3}=3x-1$ .....(i)

Squaring both sides

$$\left(\sqrt{x+3}\right)^2=(3x-1)^2$$

$$x+3=(3x)^2+(1)^2-2(3x)(1)$$

$$x+3=9x^2+1-6x$$

$$9x^2-6x-x+1-3=0$$

$$9x^2-7x-2=0$$

$$9x^2-9x+2x-2=0$$

$$9x(x-1)+2(x-1)=0$$

$$(x-1)(9x+2)=0$$

$x-1=0$ or $9x+2=0$

$x=1$ or $9x=-2$

or $x=\frac{-2}{9}$

Checking:

Putting $x=1$ in the equation (i) we have

$$\sqrt{x+3}=3x-1$$

$$\sqrt{1+3}=3(1)-1$$

$$\sqrt{4}=3-1$$

$2=2$ which is true

Putting $x=\frac{-2}{9}$ in the equation (i) we have

$$\sqrt{x+3}=3x-1$$

$$\sqrt{\frac{-2}{9}+3}=3\left(\frac{-2}{9}\right)-1$$

$$\sqrt{\frac{-2+27}{9}}=\frac{-6}{9}-1$$

$$\sqrt{\frac{25}{9}}=\frac{-6-9}{9}$$

$$\frac{5}{3}=\frac{-\cancel{15}^{5}}{\cancel{9}_{3}}$$

$$\frac{5}{3}=\frac{-5}{3}$$ which is not true

As $\frac{-2}{9}$ is an extraneous root

So, the solution set is $\{1\}$

Q.3 $4x = \sqrt{13x+14} - 3$

Solution: $4x = \sqrt{13x+14} - 3$ .....(i)

$4x+3 = \sqrt{13x+14}$

Squaring both sides

$(4x+3)^2 = \left(\sqrt{13x+14}\right)^2$

$(4x)^2 + (3)^2 + 2(4x)(3) = 13x+14$

$16x^2 + 9 + 24x = 13x+14$

$16x^2 + 9 + 24x - 13x - 14 = 0$

$16x^2 + 11x - 5 = 0$

$16x^2 + 16x - 5x - 5 = 0$

$16x(x+1) - 5(x+1) = 0$

$(x+1)(16x-5) = 0$

$x+1 = 0$ or $16x-5 = 0$

$x = -1$ or $16x = 5$

$\boxed{x = -1}$ or $\boxed{x = \frac{5}{16}}$

Checking

Putting $x = -1$ in the equation (i) we have

$4x = \sqrt{13x+14} - 3$

$4(-1) = \sqrt{13(-1)+14} - 3$

$-4 = \sqrt{-13+14} - 3$

$-4 = \sqrt{1} - 3$

$-4 = 1 - 3$

$-4 \neq -2$

So -1 is an extraneous root

Putting $x = \frac{5}{16}$ in the equation (i) we have

$4x = \sqrt{13x+14} - 3$

$4\left(\frac{5}{16}\right) = \sqrt{13\left(\frac{5}{16}\right)+14} - 3$

$\frac{20}{16} = \sqrt{\frac{65}{16}+14} - 3$

$\frac{5}{4} = \sqrt{\frac{65+224}{16}} - 3$

$\frac{5}{4} = \sqrt{\frac{289}{16}} - 3$

$\frac{5}{4} = \frac{17}{4} - 3$

$\frac{5}{4} = \frac{17-12}{4}$

$\frac{5}{4} = \frac{5}{4}$ which is true

So, the solution set is $\left\{\frac{5}{16}\right\}$

Q.4 $\sqrt{3x+100} - x = 4$

Solution: $\sqrt{3x+100} - x = 4$.....(i)

$\sqrt{3x+100} = 4+x$

Squaring both sides

$\left(\sqrt{3x+100}\right)^2 = (4+x)^2$

$3x+100 = (4)^2 + (x)^2 + 2(4)(x)$

$3x+100 = 16 + x^2 + 8x$

$x^2 + 8x + 16 - 3x - 100 = 0$

$x^2 + 5x - 84 = 0$

$x^2 + 12x - 7x - 84 = 0$

$x(x+12) - 7(x+12) = 0$

$(x+12)(x-7) = 0$

$x+12 = 0$ or $x-7 = 0$

$x = -12$ or $x = 7$

Checking

Putting $x = -12$ in the equation (i) we have

$\sqrt{3x+100} - x = 4$

$\sqrt{3(-12)+100} - (-12) = 4$

$\sqrt{-36+100} + 12 = 4$

$\sqrt{64} + 12 = 4$

$8 + 12 = 4$

$20 \neq 4$

So -12 is an extraneous root

Putting $x = 7$ in the equation (i) we have

$\sqrt{3x+100} - x = 4$

$\sqrt{3(7)+100} - 7 = 4$

$\sqrt{21+100} - 7 = 4$

$\sqrt{121} - 7 = 4$

$11 - 7 = 4$

$4 = 4$ which is true

So, the solution set is $\{7\}$

Q.5 $\sqrt{x+5}+\sqrt{x+21}=\sqrt{x+60}$

Solution: $\sqrt{x+5}+\sqrt{x+21}=\sqrt{x+60}$ ...(i)

Squaring on both sides

$$\left(\sqrt{x+5}+\sqrt{x+21}\right)^2=\left(\sqrt{x+60}\right)^2$$

$$\left(\sqrt{x+5}\right)^2+\left(\sqrt{x+21}\right)^2+2\left(\sqrt{x+5}\right)\left(\sqrt{x+21}\right)=x+60$$

$$x+5+x+21+2\left(\sqrt{x+5}\right)\left(\sqrt{x+21}\right)=x+60$$

$$2x+26+2\sqrt{x^2+21x+5x+105}=x+60$$

$$2\sqrt{x^2+26x+105}=x+60-2x-26$$

$$2\sqrt{x^2+26x+105}=-x+34$$

$$2\sqrt{x^2+26x+105}=34-x$$

Again squaring both sides

$$\left(2\sqrt{x^2+26x+105}\right)^2=(34-x)^2$$

$$4(x^2+26x+105)=(34)^2+x^2-2(34)(x)$$

$$4x^2+104x+420=1156+x^2-68x$$

$$4x^2+104x+420-1156-x^2+68x=0$$

$$3x^2+172x-736=0$$

$$a=3\ ,\ b=172\ ,\ c=-736$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-172\pm\sqrt{(172)^2-4(3)(-736)}}{2(3)}$$

$$x=\frac{-172\pm\sqrt{29584+8832}}{6}$$

$$x=\frac{-172\pm\sqrt{38416}}{6}$$

$$x=\frac{-172\pm196}{6}$$

$$x=\frac{-172-196}{6}\text{ or }x=\frac{-172+196}{6}$$

$$x=\frac{-368}{6}\quad\text{or}\quad x=\frac{24}{6}$$

$$x=\frac{-184}{3}\quad\text{or}\quad x=4$$

Checking:

Putting $x=4$ in equation (i)

$$\sqrt{x+5}+\sqrt{x+21}=\sqrt{x+60}$$

$$\sqrt{4+5}+\sqrt{4+21}=\sqrt{4+60}$$

$$\sqrt{9}+\sqrt{25}=\sqrt{64}$$

$$3+5=8$$

$8=8$ which is true

Putting $x=\frac{-184}{3}$

$$\sqrt{x+5}+\sqrt{x+21}=\sqrt{x+60}$$

$$\sqrt{\frac{-184}{3}+5}+\sqrt{\frac{-184}{3}+21}=\sqrt{\frac{-184}{3}+60}$$

$$\sqrt{\frac{-184+15}{3}}+\sqrt{\frac{-184+63}{3}}=\sqrt{\frac{-184+180}{3}}$$

$$\sqrt{\frac{-169}{3}}+\sqrt{\frac{-121}{3}}=\sqrt{\frac{-4}{3}}$$

$$\sqrt{\frac{-1\times169}{3}}+\sqrt{\frac{-1\times121}{3}}=\sqrt{\frac{-1\times4}{3}}$$

$$\frac{13i}{\sqrt{3}}+\frac{11i}{\sqrt{3}}=\frac{2i}{\sqrt{3}}\qquad\because i=\sqrt{-1}$$

$$\frac{13i+11i}{\sqrt{3}}=\frac{2i}{\sqrt{3}}$$

$$\frac{24i}{\sqrt{3}}=\frac{2i}{\sqrt{3}}$$ Which is not true

As $x=\frac{-184}{3}$ is extraneous root

so solution set is $\{4\}$

Q.6 $\sqrt{x+1}+\sqrt{x-2}=\sqrt{x+6}$

Solution: $\sqrt{x+1}+\sqrt{x-2}=\sqrt{x+6}$ .....(i)

Squaring both sides

$$\left(\sqrt{x+1}+\sqrt{x-2}\right)^2=\left(\sqrt{x+6}\right)^2$$

$$\left(\sqrt{x+1}\right)^2+\left(\sqrt{x-2}\right)^2+2\left(\sqrt{x+1}\right)\left(\sqrt{x-2}\right)=x+6$$

$$x+1+x-2+2\sqrt{(x+1)(x-2)}=x+6$$

$$2x-1+2\sqrt{x^2-2x+1x-2}=x+6$$

$$2\sqrt{x^2-x-2}=x+6-2x+1$$

$$2\sqrt{x^2-x-2}=7-x$$

Again squaring both sides

$$\left(2\sqrt{x^2-x-2}\right)^2=(7-x)^2$$

$$4(x^2-x-2)=(7)^2+(x)^2-2(7)(x)$$

$$4x^2-4x-8=49+x^2-14x$$

$$4x^2-x^2-4x+14x-8-49=0$$

$$3x^2+10x-57=0$$

$$3x^2+19x-9x-57=0$$

$$x(3x+19)-3(3x+19)=0$$

$$(3x+19)(x-3)=0$$

$3x+19=0$ or $x-3=0$

$3x=-19$ or $x=3$

$$x=\frac{-19}{3}$$

Checking

Putting $x=3$ in the equation (i)

$$\sqrt{x+1}+\sqrt{x-2}=\sqrt{x+6}$$

$$\sqrt{3+1}+\sqrt{3-2}=\sqrt{3+6}$$

$$\sqrt{4}+\sqrt{1}=\sqrt{9}$$

$$2+1=3$$

$3=3$ which is true

Putting $x=\frac{-19}{3}$ in the equation (i)

$$\sqrt{x+1}+\sqrt{x-2}=\sqrt{x+6}$$

$$\sqrt{\frac{-19}{3}+1}+\sqrt{\frac{-19}{3}-2}=\sqrt{\frac{-19}{3}+6}$$

$$\sqrt{\frac{-19+3}{3}}+\sqrt{\frac{-19-6}{3}}=\sqrt{\frac{-19+18}{3}}$$

$$\sqrt{\frac{-1\times16}{3}}+\sqrt{\frac{-1\times25}{3}}=\sqrt{\frac{-1\times1}{3}}$$

$$\frac{4i}{\sqrt{3}}+\frac{5i}{\sqrt{3}}=\frac{1i}{\sqrt{3}} \qquad \because i=\sqrt{-1}$$

$$\frac{4i+5i}{\sqrt{3}}=\frac{1i}{\sqrt{3}}$$

$\frac{9i}{\sqrt{3}}=\frac{1i}{\sqrt{3}}$ which is not true

As $x=\frac{-19}{3}$ is an extraneous root

So the solution set is $\{3\}$

Q.7 $\sqrt{11-x}+\sqrt{6-x}=\sqrt{27-x}$

Solution: $\sqrt{11-x}+\sqrt{6-x}=\sqrt{27-x}$ .....(i)

Squaring both sides

$$\left(\sqrt{11-x}+\sqrt{6-x}\right)^2=\left(\sqrt{27-x}\right)^2$$

$$\left(\sqrt{11-x}\right)^2+\left(\sqrt{6-x}\right)^2+2\left(\sqrt{11-x}\right)\left(\sqrt{6-x}\right)=27-x$$

$$11-x+6-x+2\sqrt{(11-x)(6-x)}=27-x$$

$$17-2x+2\sqrt{66-11x-6x+x^2}=27-x$$

$$2\sqrt{66-17x+x^2}=27-x-17+2x$$

$$2\sqrt{66-17x+x^2}=10+x$$

Again squaring both sides

$$\left(2\sqrt{66-17x+x^2}\right)^2=(10+x)^2$$

$$4(66-17x+x^2)=(10)^2+(x)^2+2(10)(x)$$

$$264-68x+4x^2=100+x^2+20x$$

$$4x^2-x^2-68x-20x+264-100=0$$

$$3x^2-88x+164=0$$

By applying quadratic formula

$a=3$, $b=-88$, $c=164$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-88)\pm\sqrt{(-88)^2-4(3)(164)}}{2(3)}$$

$$x=\frac{88\pm\sqrt{7744-1968}}{6}$$

$$x=\frac{88\pm\sqrt{5776}}{6}$$

$$x=\frac{88-76}{6} \quad \text{or} \quad x=\frac{88+76}{6}$$

$$x=\frac{\cancel{12}^{\,2}}{\cancel{6}} \quad \text{or} \quad x=\frac{\cancel{164}^{\,82}}{\cancel{6}_{\,3}}$$

$x=2$ or $x=\frac{82}{3}$

Checking

Putting $x = 2$ in the equation (i)

$$\sqrt{11-x}+\sqrt{6-x}=\sqrt{27-x}$$

$$\sqrt{11-2}+\sqrt{6-2}=\sqrt{27-2}$$

$$\sqrt{9}+\sqrt{4}=\sqrt{25}$$

$$3+2=5$$

$5 = 5$ which is true

Putting $x=\frac{82}{3}$ in the equation (i)

$$\sqrt{11-x}+\sqrt{6-x}=\sqrt{27-x}$$

$$\sqrt{11-\frac{82}{3}}+\sqrt{6-\frac{82}{3}}=\sqrt{27-\frac{82}{3}}$$

$$\sqrt{\frac{33-82}{3}}+\sqrt{\frac{18-82}{3}}=\sqrt{\frac{81-82}{3}}$$

$$\sqrt{\frac{-1\times 49}{3}}+\sqrt{\frac{-1\times 64}{3}}=\sqrt{\frac{-1\times 1}{3}}$$

$$\frac{7i}{\sqrt{3}}+\frac{8i}{\sqrt{3}}=\frac{i}{\sqrt{3}} \qquad \because i=\sqrt{-1}$$

$$\frac{7i+8i}{\sqrt{3}}=\frac{i}{\sqrt{3}}$$

$$\frac{15i}{\sqrt{3}}=\frac{i}{\sqrt{3}}$$ which is not true

As $\frac{82}{3}$ is an extraneous root

So, the solution set is $\{2\}$

Q.8 $\sqrt{4a+x}-\sqrt{a-x}=\sqrt{a}$

Solution: $\sqrt{4a+x}-\sqrt{a-x}=\sqrt{a}$ .....(i)

Squaring both sides

$$\left(\sqrt{4a+x}-\sqrt{a-x}\right)^2=\left(\sqrt{a}\right)^2$$

$$\left(\sqrt{4a+x}\right)^2+\left(\sqrt{a-x}\right)^2-2\left(\sqrt{4a+x}\right)\left(\sqrt{a-x}\right)=a$$

$$4a+\cancel{x}+a-\cancel{x}-2\sqrt{(4a+x)(a-x)}=a$$

$$5a-2\sqrt{4a^2-4ax+ax-x^2}=a$$

$$-2\sqrt{4a^2-3ax-x^2}=a-5a$$

$$-2\sqrt{4a^2-3ax-x^2}=-4a$$

$$\sqrt{4a^2-3ax-x^2}=\frac{-\cancel{4}^{2}a}{-\cancel{2}}$$

$$\sqrt{4a^2-3ax-x^2}=2a$$

Again squaring both sides

$$\left(\sqrt{4a^2-3ax-x^2}\right)^2=(2a)^2$$

$$4a^2-3ax-x^2=4a^2$$

$$4a^2-3ax-x^2=4a^2$$

$$4\cancel{a^2}-4\cancel{a^2}=3ax+x^2$$

$\Rightarrow$ $3ax+x^2=0$

$$x(3a+x)=0$$

$x = 0$ or $3a+x=0$

$x = 0$ or $x=-3a$

Checking:

Putting $x = 0$ in the equation (i)

$$\sqrt{4a+x}-\sqrt{a-x}=\sqrt{a}$$

$$\sqrt{4a+0}-\sqrt{a-0}=\sqrt{a}$$

$$\sqrt{4a}-\sqrt{a}=\sqrt{a}$$

$$2\sqrt{a}-\sqrt{a}=\sqrt{a}$$

$\sqrt{a}=\sqrt{a}$ which is true

Putting $x=-3a$ in the equation (i)

$$\sqrt{4a+x}-\sqrt{a-x}=\sqrt{a}$$

$$\sqrt{4a-3a}-\sqrt{a+3a}=\sqrt{a}$$

$$\sqrt{a}-\sqrt{4a}=\sqrt{a}$$

$$\sqrt{a}-2\sqrt{a}=\sqrt{a}$$

$-\sqrt{a}=\sqrt{a}$ which is not true

As $-3a$ is extraneous root

So, the solution set is $\{0\}$

Q.9 $\sqrt{x^2+x+1}-\sqrt{x^2+x-1}=1$

Solution: $\sqrt{x^2+x+1}-\sqrt{x^2+x-1}=1$ .....(i)

Let $x^2+x=y$ .....(ii)

Put it in equation (i)

$$\sqrt{y+1}-\sqrt{y-1}=1$$

Squaring both sides

$$\left(\sqrt{y+1}-\sqrt{y-1}\right)^2=(1)^2$$

$$\left(\sqrt{y+1}\right)^2+\left(\sqrt{y-1}\right)^2-2\left(\sqrt{y+1}\right)\left(\sqrt{y-1}\right)=1$$

$$y+1+y-1-2\sqrt{(y+1)(y-1)}=1$$

$$2y-2\sqrt{y^2-1}=1$$

$$\Rightarrow \quad 2y-1=2\sqrt{y^2-1}$$

Again squaring both sides

$$(2y-1)^2=\left(2\sqrt{y^2-1}\right)^2$$

$$(2y)^2+(1)^2-2(2y)(1)=4\left(y^2-1\right)$$

$$4y^2+1-4y=4y^2-4$$

$$4y^2+1-4y=4y^2-4$$

$$1-4y=-4$$

$$1+4=4y$$

$$5=4y$$

$$\Rightarrow \quad 4y=5$$

From equation (ii) Put $y=x^2+x$

$$4y=5$$

$$4(x^2+x)=5$$

$$4x^2+4x=5$$

$$4x^2+4x-5=0$$

Solving by quadratic formula

$a=4$, $b=4$, $c=-5$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-4\pm\sqrt{(4)^2-4(4)(-5)}}{2(4)}$$

$$x=\frac{-4\pm\sqrt{16+80}}{8}$$

$$x=\frac{-4\pm\sqrt{96}}{8}$$

$$x=\frac{-4\pm\sqrt{16\times6}}{8}$$

$$x=\frac{-4\pm4\sqrt{6}}{8}$$

$$x=\frac{\cancel{4}\left(-1\pm\sqrt{6}\right)}{\cancel{8}_2}$$

$$x=\frac{-1\pm\sqrt{6}}{2}$$

The solution set is $\left\{\frac{-1\pm\sqrt{6}}{2}\right\}$

Q.10 $\sqrt{x^2+3x+8}+\sqrt{x^2+3x+2}=3$

Solution: $\sqrt{x^2+3x+8}+\sqrt{x^2+3x+2}=3$ ....(i)

Let $x^2+3x=y$ .....(ii)

Put it in equation (i)

$$\sqrt{y+8}+\sqrt{y+2}=3$$

Squaring on both sides

$$\left(\sqrt{y+8}+\sqrt{y+2}\right)^2=(3)^2$$

$$\left(\sqrt{y+8}\right)^2+\left(\sqrt{y+2}\right)^2+2\left(\sqrt{y+8}\right)\left(\sqrt{y+2}\right)=9$$

$$y+8+y+2+2\sqrt{y^2+2y+8y+16}=9$$

$$2y+10+2\sqrt{y^2+10y+16}=9$$

$$2\sqrt{y^2+10y+16}=9-2y-10$$

$$2\sqrt{y^2+10y+16}=-2y-1$$

$$2\sqrt{y^2+10y+16}=-(2y+1)$$

Again squaring on both sides

$$\left(2\sqrt{y^2+10y+16}\right)^2=\left[-(2y+1)\right]^2$$

$$4(y^2+10y+16)=(2y+1)^2$$

$4y^2+40y+64=(2y)^2+(1)^2+2(2y)(1)$

$4y^2+40y+64=4y^2+1+4y$

$40y+64=1+4y$

$40y-4y=1-64$

$36y=-63$

From equation (ii) Put $y=x^2+3x$

$36(x^2+3x)=-63$

$36x^2+108x+63=0$

Solving by quadratic formula

$a=36\ ,\ b=108\ ,\ c=63$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-108\pm\sqrt{(108)^2-4(36)(63)}}{2(36)}$$

$$=\frac{-108\pm\sqrt{11664-9072}}{72}$$

$$=\frac{-108\pm\sqrt{2592}}{72}$$

$$x=\frac{-108\pm\sqrt{4\times4\times9\times9\times2}}{72}$$

$$x=\frac{-108\pm\sqrt{4^2\times9^2\times2}}{72}$$

$$x=\frac{-108\pm4\times9\sqrt{2}}{72}$$

$$x=\frac{-108\pm36\sqrt{2}}{72}$$

$$x=\frac{\cancel{36}\left(-3\pm\sqrt{2}\right)}{\cancel{72}_2}$$

$$x=\frac{-3\pm\sqrt{2}}{2}$$

So, the solution set is $\left\{\frac{-3\pm\sqrt{2}}{2}\right\}$

Q.11 $\sqrt{x^2+3x+9}+\sqrt{x^2+3x+4}=5$

Solution: $\sqrt{x^2+3x+9}+\sqrt{x^2+3x+4}=5$ ....(i)

Let $x^2+3x=y$ .....(ii)

Put it in equation (i)

$$\sqrt{y+9}+\sqrt{y+4}=5$$

Squaring both sides

$$\left(\sqrt{y+9}+\sqrt{y+4}\right)^2=5^2$$

$$\left(\sqrt{y+9}\right)^2+\left(\sqrt{y+4}\right)^2+2\left(\sqrt{y+9}\right)\left(\sqrt{y+4}\right)=25$$

$$y+9+y+4+2\sqrt{(y+9)(y+4)}=25$$

$$2y+13+2\sqrt{y^2+4y+9y+36}=25$$

$$2\sqrt{y^2+13y+36}=25-2y-13$$

$$2\sqrt{y^2+13y+36}=12-2y$$

$$\cancel{2}\sqrt{y^2+13y+36}=\cancel{2}(6-y)$$

$$\sqrt{y^2+13y+36}=6-y$$

Again taking squaring both sides

$$\left(\sqrt{y^2+13y+36}\right)^2=(6-y)^2$$

$$y^2+13y+36=(6)^2+(y)^2-2(6)(y)$$

$$\cancel{y^2}+13y+\cancel{36}=\cancel{36}+\cancel{y^2}-12y$$

$$13y=-12y$$

$$13y+12y=0$$

$$25y=0$$

$$y=\frac{0}{25}$$

$$y=0$$

From equation (ii) Put $x^2+3x=y$

$$x^2+3x=0$$

$$x(x+3)=0$$

$x=0$ or $x+3=0$

$x=-3$

So, the solution set is $\{-3,\ 0\}$

MISCELLANEOUS EXERCISE – 1

Q. 1 Multiple Choice Questions

Four possible answers are given for the following questions. Tick (✓) the correct answer.

1. An equation, which remains unchanged when x is replaced by $\frac{1}{x}$ is called a/an.....
   (a) Exponential equation
   (b) Reciprocal equation
   (c) Radical equation
   (d) None of these

2. The solution set of equation $4x^2-16=0$ is
   (a) $\{\pm 4\}$ (b) $\{4\}$
   (c) $\{\pm 2\}$ (d) ± 2

3. The number of methods to solve a quadratic equation is
   (a) 1 (b) 2
   (c) 3 (d) 4

4. An equation of the form $2^{2x}-3.2^{x}+5=0$ is called a / an ___ equation.
   (a) Exponential (b) Radical
   (c) Reciprocal (d) None of these

5. Two linear factors of $x^2-15x+56$ are
   (a) $(x-7)$ and $(x+8)$
   (b) $(x+7)$ and $(x-8)$
   (c) $(x-7)$ and $(x-8)$
   (d) $(x+7)$ and $(x+8)$

6. Standard form of quadratic equation is _____
   (a) $bx+c=0,\ b\neq 0$
   (b) $ax^2+bx+c=0,\ a\neq 0$
   (c) $ax^2=bx,\ a\neq 0$
   (d) $ax^2=0,\ a\neq 0$

7. The solution set of the equation $x^2-9=0$ is
   (a) $\{\pm 3\}$ (b) $\{3\}$
   (c) $\{-3\}$ (d) $\{9\}$

8. The number of terms in a standard quadratic equation $ax^2+bx+c=0$ is ___
   (a) 1 (b) 2
   (c) 3 (d) 4

9. The solution set of equation $2+9x=5x^2$ is.....
   (a) $\left\{\frac{-1}{5},2\right\}$ (b) $\left\{\frac{+1}{5},2\right\}$
   (c) $\left\{\frac{1}{5},-2\right\}$ (d) $\left\{\frac{-1}{5},-2\right\}$

10. The solution set of equation $x^2-16=0$ is.....
    (a) $\{\pm 4\}$ (b) $\{+4\}$
    (c) $\{-4\}$ (d) None of these

11. The quadratic formula is
    (a) $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ (b) $\frac{b\pm\sqrt{b^2-4ac}}{2a}$
    (c) $\frac{-b\pm\sqrt{b^2+4ac}}{2a}$ (d) $\frac{b\pm\sqrt{b^2+4ac}}{2a}$

12. An equation of the type $3^x+3^{2-x}+6=0$ is a/an.....
    (a) Exponential equation
    (b) Reciprocal equation
    (c) Radical equation
    (d) None of these

13. If $a=0$, in $ax^2+bx+c=0$, then it reduces to.....
    (a) pure quadratic equation
    (b) linear equation
    (c) quadratic equations
    (d) exponential equation

14. How many linear factors a quadratic equation has?
    (a) 1 (b) 2
    (c) 3 (d) 4

15. An equation of the form $2x^4-3x^3+7x^2-3x+2=0$ is called a/an
    (a) Reciprocal equation
    (b) Radical equation
    (c) Exponential equation
    (d) None of these

16. The solution set of equation $5x^2=30x$ is.....
    (a) $\{5,30\}$ (b) $\{0,6\}$
    (c) $\{0,-6\}$ (d) $\{5,0\}$

17. The standard form of the quadratic equation is $ax^2+bx+c=0$ where a, b, c are .....
(a) Irrational numbers
(b) Rational numbers
(c) Real numbers
(d) Whole numbers

18. The solution set of $25x^2-1=0$ is
(a) $\left\{\pm\frac{1}{5}\right\}$ (b) $\left\{-\frac{1}{5}\right\}$
(c) $\left\{+\frac{1}{5}\right\}$ (d) None of these

19. Cancellation of x on both sides of $5x^2=30x$ means
(a) the loss of one root
(b) no loss of any root
(c) gain of one root
(d) undefined solution

20. If $2^x=1$, then x = ...
(a) 0 (b) 1
(c) 2 (d) $\frac{1}{2}$

21. What should be done to make the co-efficient of x^2 equal to 1 in $3x^2+7x=0$?
(a) multiply the equation by $\frac{1}{3}$
(b) divide the equation by $\frac{1}{3}$
(c) add $\frac{1}{3}$ in both sides
(d) subtract $\frac{1}{3}$ from both sides

22. Sentences involving the Sign..... between two algebraic expressions are called equations
(a) $<$ (b) $\geq$
(c) $=$ (d) $<$ or $>$

23. The solution set of equation $x^2-x-2=0$ is.....
(a) {2, 1} (b) {−2,1}
(c) {2, −1} (d) {−2, −1}

24. An equation of type $x^4+x^3+x^2+x+1=0$ is called a/ an .....equation.
(a) Radical (b) Reciprocal
(c) Exponential (d) None of these

25. Solve the equation $5^{1+x}+5^{1-x}=26$
(a) {1} (b) {±1}
(c) {2} (d) {±2}

26. The value of variable of an equation not satisfying the equation is called.
(a) root (b) extraneous root
(c) exponent (d) solution set

27. The number of roots of a quadratic equation is:
(a) 1 (b) 2
(c) 3 (d) 4

28. The solution set of equation $x^2-7x+6=0$ is.....
(a) {1, 6} (b) {−1, −6}
(c) {−1, 6} (d) { 1, −6}

29. What should be done to make the co-efficient of x^2 equal to 1, in $7x^2+2x-1=0$?
(a) multiply the equation by 7
(b) divide the equation by 7
(c) add 7 in both sides
(d) subtract 7 from both sides

30. If b=0 in a quadratic equation $ax^2+bx+c=0$, then it is called.
(a) Pure quadratic equation
(b) Linear equation
(c) Quadratic equation
(d) Exponential equation

31. What is the degree of quadratic equation?
(a) 1 (b) 2
(c) 3 (d) 4

32. The cancellation of x on both sides of the equation of the type $ax^2=bx$ means the loss of one root. That root is always equal to.....
(a) 0 (b) 1
(c) a (d) b

33. If $y=2^x$ and 8y =1, then, x =...
(a) 8 (b) $\frac{1}{8}$

(c) 3 (d) -3

34. The solution set of equation $3x^2+4x=5$ is……..

(a) $\left\{\frac{-2\pm\sqrt{19}}{3}\right\}$ (b) $\left\{\frac{2\pm\sqrt{19}}{3}\right\}$

(c) $\left\{\frac{4\pm\sqrt{19}}{3}\right\}$ (d) None of these

35. If $y=x^{-1}$ and $3y=5$, the value of x is ….

(a) $\frac{5}{3}$ (b) $\frac{-5}{3}$

(c) $\frac{-3}{5}$ (d) $\frac{3}{5}$

ANSWER KEY

1.	b	**2.**	c	**3.**	c	**4.**	a	**5.**	c
6.	b	**7.**	a	**8.**	c	**9.**	a	**10.**	a
11.	a	**12.**	a	**13.**	b	**14.**	b	**15.**	a
16.	b	**17.**	c	**18.**	a	**19.**	a	**20.**	a
21.	a	**22.**	c	**23.**	c	**24.**	b	**25.**	b
26.	b	**27.**	b	**28.**	a	**29.**	b	**30.**	a
31.	b	**32.**	a	**33.**	d	**34.**	a	**35.**	d

Q.2. Write short answers of the following questions.

(i) Solve $x^2 + 2x - 2 = 0$

Solution: $x^2 + 2x - 2 = 0$

$ax^2 + bx + c = 0$

$\Rightarrow$ $a = 1$, $b = 2$, $c = -2$

We know that

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-2\pm\sqrt{(2)^2-4(1)(-2)}}{2(1)}$$

$$x=\frac{-2\pm\sqrt{4+8}}{2}$$

$$x=\frac{-2\pm\sqrt{12}}{2}$$

$$x=\frac{-2\pm\sqrt{4\times3}}{2}$$

$$x=\frac{-2\pm2\sqrt{3}}{2}$$

$$x=\frac{\cancel{2}(-1\pm\sqrt{3})}{\cancel{2}}$$

$$x=-1\pm\sqrt{3}$$

$$S.S=\left\{-1\pm\sqrt{3}\right\}$$

(ii) Solve by factorization $5x^2 = 15x$

Solution:

$5x^2 = 15x$

$5x^2 - 15x = 0$

$5x(x - 3) = 0$

$5x(x - 3) = 0$

$5x = 0$ or $x - 3 = 0$

$x=\frac{0}{5}$ or $x = 3$

$x = 0$ or $x = 3$

Solution set is $\{0, 3\}$

(iii) Write in standard form $\frac{1}{x+4}+\frac{1}{x-4}=3$

Solution: $\frac{1}{x+4}+\frac{1}{x-4}=3$

$$\frac{(x-4)+(x+4)}{(x+4)(x-4)}=3$$

$$\frac{x-\cancel{4}+x+\cancel{4}}{(x)^2-(4)^2}=3$$

$$\frac{2x}{x^2-16}=3$$

$2x = 3(x^2 - 16)$

$2x = 3x^2 - 48$

$0 = 3x^2 - 2x - 48$

$3x^2 - 2x - 48 = 0$

(iv) Write the name of methods for solving a quadratic equation.

Ans. Following three methods are used for solving the quadratic equation.

(i) Factorization method

(ii) Completing square method

(iii) Quadratic formula method

(v) **Solve** $\left(2x-\frac{1}{2}\right)^2=\frac{9}{4}$

Solution:

$$\left(2x-\frac{1}{2}\right)^2=\frac{9}{4}$$

Taking square root

$$\sqrt{\left(2x-\frac{1}{2}\right)^2}=\pm\sqrt{\frac{9}{4}}$$

$$2x-\frac{1}{2}=\pm\frac{3}{2}$$

$$2x=\pm\frac{3}{2}+\frac{1}{2}$$

$$2x=\frac{\pm3+1}{2}$$

$$x=\frac{\pm3+1}{2\times2}$$

$$x=\frac{\pm3+1}{4}$$

$\Rightarrow$ $x=\frac{3+1}{4}$ or $x=\frac{-3+1}{4}$

$x=\frac{4}{4}$ or $x=\frac{-2}{4}$

$x=1$ or $x=\frac{-1}{2}$

The Solution set is $\left\{1,\frac{-1}{2}\right\}$

(vi) **Solve** $\sqrt{3x+18}=x$

Solution:

$\sqrt{3x+18}=x$ .....(i)

Taking square of both sides

$(\sqrt{3x+18})^2=(x)^2$

$3x+18=x^2$

$\Rightarrow$ $x^2-3x-18=0$

$x^2-6x+3x-18=0$

$x(x-6)+3(x-6)=0$

$(x-6)(x+3)=0$

$x-6=0$ or $x+3=0$

$x=6$ or $x=-3$

Checking

Put $x=6$ in equation (i)

$\sqrt{3x+18}=x$

$\sqrt{3(6)+18}=6$

$\sqrt{18+18}=6$

$\sqrt{36}=6$

$6=6$ which is true

Put $x=-3$ in given equation

$\sqrt{3x+18}=x$

$\sqrt{3(-3)+18}=-3$

$\sqrt{-9+18}=-3$

$\sqrt{9}=-3$

$3=-3$ which is not true

As -3 is an extraneous root

So, the solution set is $\{6\}$

(vii) Define quadratic equation

Quadratic Equation

An equation which contains the square of the unknown (variable) quantity, but no higher power, is called a quadratic equation. Standard form of quadratic equation in one variable is $ax^2+bx+c=0$, where $a, b, c \in R$ and $a \neq 0$.

(viii) Define reciprocal equation

Reciprocal Equation

An equation is said to be a reciprocal equation, if it remains unchanged, when x is replaced by $\frac{1}{x}$.

Example

(i) $ax^4+bx^3+cx^2+bx+a=0$ OR

(ii) $a\left(x^2+\frac{1}{x^2}\right)+b\left(x+\frac{1}{x}\right)+c=0$

(ix) Define exponential equation:

Exponential Equation:

An equation in which variable occurs in exponent is called exponential equation.

Example: $2^x-8=0$

(x) **Define radical equation**

Radical Equation

An equation involving expression of variable under the radical sign is called radical equation.

Examples: (i) $\sqrt{x+3}=1$

(ii) $\sqrt{x-1}=\sqrt{x-2}$

Q.3 Fill in the blanks:

i. The standard form of the quadratic equation is_____.

ii. The number of methods to solve a quadratic equation are.....

iii. The name of the method to derive a quadratic formula is .....

iv. The solution of the equation $ax^2+bx+c=0$, $a \neq 0$_____.

v. The solution set of $25x^2-1=0$ is_____.

vi. An equation of the form $2^{2x}-3.2^x+5=0$ is called a/an _____equation.

vii. The solution set of the equation $x^2-9=0$ is_____.

viii. An equation of the type $x^4+x^3+x^2+x+1=0$ is called an_____ equation.

ix. A root of an equation, which do not satisfy the equation is called _____root.

x. An equation involving expression of the variable under_____ is called radical equation.

Answers:

i. $ax^2 + bx + c = 0, \quad a \neq 0$

ii. Three

iii. Completing square

iv. $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

v. $\left\{\pm\dfrac{1}{5}\right\}$

vi. Exponential

vii. $\{\pm 3\}$

viii. Reciprocal

ix. Extraneous

x. Radical Sign