

www.ustad360.com

CHAPTER

11

TRIGONOMETRIC FUNCTIONS & THEIR GRAPHS

DOMAIN AND RANGE OF TRIGONOMETRIC FUNCTION

Function	Domain	Range
$y = \sin x$	$\mathbb{R}$	$-1 \leq y \leq 1$
$y = \cos x$	$\mathbb{R}$	$-1 \leq y \leq 1$
$y = \tan x$	$\forall x \in \mathbb{R}, \text{ but } x \neq (2n+1)\frac{\pi}{2}, n \in \mathbb{Z}$	$\mathbb{R}$
$y = \cot x$	$\forall x \in \mathbb{R}, \text{ but } x \neq n\pi, n \in \mathbb{Z}$	$\mathbb{R}$
$y = \sec x$	$\forall x \in \mathbb{R}, \text{ but } x \neq (2n+1)\frac{\pi}{2}, n \in \mathbb{Z}$	$y \geq 1 \text{ or } y \leq -1$
$y = \operatorname{cosec} x$	$\forall x \in \mathbb{R}, \text{ but } x \neq n\pi, n \in \mathbb{Z}$	$y \geq 1 \text{ or } y \leq -1$

Period

Period of a trigonometric function is the smallest +ve number which, when added to the original circular measure of the angle, given the same value of the function.

Formula to Find Period

$$f(x) = f(x + P)$$

Standard Periods

$$\sin x = 2\pi$$

$$\tan x = \pi$$

$$\cos x = 2\pi$$

$$\cot x = \pi$$

$$\sec x = 2\pi$$

$$\operatorname{cosec} x = 2\pi$$

www.ustad360.com

EXERCISE 11.1

Find the periods of the following functions:

Q.1 $\sin 3x$ (Lahore Board 2005)

Solution:

Given function is $\sin 3x$

Since period of $\sin$ is 2π

Therefore $f(x) = f(x + P)$

$$\sin 3x = \sin (3x + 2\pi)$$

$$\sin 3x = \sin 3\left(x + \frac{2\pi}{3}\right)$$

Thus period of $\sin 3x$ is $\frac{2\pi}{3}$

Q.2 $\cos 2x$

Solution:

Given function is $\cos 2x$

Since period of $\cos$ is 2π

Therefore $f(x) = f(x + P)$

$$\cos 2x = \cos (2x + 2\pi)$$

$$\cos 2x = \cos 2(x + \pi)$$

Thus period of $\cos 2x$ is π .

Q.3 $\tan 4x$ (Gujranwala Board 2007)

Solution:

Given function is $\tan 4x$

Since period of $\tan x$ is π

Therefore $f(x) = f(x + P)$

$$\tan 4x = \tan (4x + \pi)$$

$$\tan 4x = \tan 4\left(x + \frac{\pi}{4}\right)$$

Thus period of $\tan 4x$ is $\frac{\pi}{4}$.



Q.4 $\cot \frac{x}{2}$ **Solution:**

Given function is $\cot \frac{x}{2}$

Since period of $\cot$ is π

Therefore $f(x) = f(x + P)$

$$\cot \frac{x}{2} = \cot \left(\frac{x}{2} + \pi \right)$$

$$\cot \frac{x}{2} = \cot \frac{1}{2} (x + 2\pi)$$

Thus period of $\cot \frac{x}{2}$ is 2π

Q.5 $\sin \frac{x}{3}$ (Lahore Board 2006)**Solution:**

Given function is $\sin \frac{x}{3}$

Since period of $\sin$ is 2π

Therefore $f(x) = f(x + P)$

$$\sin \frac{x}{3} = \sin \left(\frac{x}{3} + 2\pi \right)$$

$$\sin \frac{x}{3} = \sin \frac{1}{3} (x + 6\pi)$$

Thus period of $\sin \frac{x}{3}$ is 6π

Q.6 $\operatorname{cosec} \frac{x}{4}$ **Solution:**

Given function is $\operatorname{cosec} \frac{x}{4}$

Since period of cosec is 2π

Therefore $f(x) = f(x + P)$

$$\operatorname{cosec} \frac{x}{4} = \operatorname{cosec} \left(\frac{x}{4} + 2\pi \right)$$



$$\operatorname{cosec} \frac{x}{4} = \operatorname{cosec} \frac{1}{4} (x + 8\pi)$$

Thus period of $\operatorname{cosec} \frac{x}{4}$ is 8π

Q.7 $\sin \frac{x}{5}$ (Lahore Board 2010)

Solution:

Given function is $\sin \frac{x}{5}$

Since period of $\sin x$ is 2π

Therefore $f(x) = f(x + P)$

$$\sin \frac{x}{5} = \sin \left(\frac{x}{5} + 2\pi \right)$$

$$\sin \frac{x}{5} = \sin \frac{1}{5} (x + 10\pi)$$

Thus period of $\sin \frac{x}{5}$ is 10π

Q.8 $\cos \frac{x}{6}$

Solution:

Given function is $\cos \frac{x}{6}$

Since period of $\cos x$ is 2π

$f(x) = f(x + P)$

$$\text{Therefore } \cos \frac{x}{6} = \cos \left(\frac{x}{6} + 2\pi \right) = \cos \frac{1}{6} (x + 12\pi)$$

Thus period of $\cos \frac{x}{6}$ is 12π

Q.9 $\tan \frac{x}{7}$

Solution:

Given function is $\tan \frac{x}{7}$

Since period of $\tan x$ is π



$$f(x) = f(x + P)$$

$$\text{Therefore } \tan \frac{x}{7} = \tan \left(\frac{x}{7} + \pi \right)$$

$$\tan \frac{x}{7} = \tan \frac{1}{7} (x + 7\pi)$$

Thus period of $\tan \frac{x}{7}$ is 7π

Q.10 $\cot 8x$

Solution:

Given function is $\cot 8x$

Since period of $\cot x$ is π

$$f(x) = f(x + P)$$

$$\text{Therefore } \cot 8x = \cot (8x + \pi)$$

$$\cot 8x = \cot 8 \left(x + \frac{\pi}{8} \right)$$

Thus period of $\cot 8x$ is $\frac{\pi}{8}$

Q.11 $\sec 9x$

Solution:

Given function is $\sec 9x$

Since period of $\sec x$ is 2π

$$f(x) = f(x + P)$$

$$\text{Therefore } \sec 9x = \sec (9x + 2\pi)$$

$$\sec 9x = \sec 9 \left(x + \frac{2\pi}{9} \right)$$

Thus period of $\sec 9x$ is $\frac{2\pi}{9}$

Q.12 $\operatorname{cosec} 10x$

Solution:

Given function is $\operatorname{cosec} 10x$

Since period of $\operatorname{cosec} x$ is 2π

$$f(x) = f(x + P)$$



Therefore $\operatorname{cosec} 10x = \operatorname{cosec} (10x + 2\pi)$

$$\operatorname{cosec} 10x = \operatorname{cosec} 10 \left(x + \frac{2\pi}{10} \right)$$

Thus period of $\operatorname{cosec} 10x$ is $\frac{\pi}{5}$

Q.13 $3 \sin x$

Solution:

Given function is $3 \sin x$

Since period of $\sin x$ is 2π

$$f(x) = f(x + P)$$

$$\text{Therefore } 3 \sin x = 3 \sin (x + 2\pi)$$

Thus period of $3 \sin x$ is 2π .

Q.14 $2 \cos x$

Solution:

Given function is $2 \cos x$

Since period of $\cos x$ is 2π

$$f(x) = f(x + P)$$

$$\text{Therefore } 2 \cos x = 2 \cos (x + 2\pi)$$

Thus period of $2 \cos x$ is 2π

Q.15 $3 \cos \frac{x}{5}$ (Lahore Board 2008)

Solution:

Given function is $3 \cos \frac{x}{5}$

Since period of $\cos x$ is 2π

$$f(x) = f(x + P)$$

$$\text{Therefore } 3 \cos \frac{x}{5} = 3 \cos \left(\frac{x}{5} + 2\pi \right)$$

$$3 \cos \frac{x}{5} = 3 \cos \frac{1}{5} (x + 10\pi)$$

Thus period of $3 \cos \frac{x}{5}$ is 10π .



EXERCISE 11.2

Q.1 Draw the graph of each of the following equations for the intervals mentioned against each:

(i) $y = -\sin x$; $x \in [-2\pi, 2\pi]$

Solution:

Given function is $y = -\sin x$

We want to draw the graph of $y = -\sin x$ from -360 to 360° .

Taking the sub-interval of 30° , we have the following table of values.

x	-360°	-330°	-300°	-270°	-240°	-210°	-180°	-150°	-120°	-90°	-60°	-30°
y	0	-0.5	-0.9	-1	-0.9	-0.5	0	0.5	0.9	1	0.9	0.5

x	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
y	0	-0.5	0.9	-1	-0.9	-0.5	0	0.5	0.9	1	0.9	0.5	0

Pairs

$(-360^\circ, 0)$, $(-330^\circ, -0.5)$, $(-300^\circ, -0.9)$, $(-270^\circ, -1)$, $(-240^\circ, -0.9)$

$(-210^\circ, -0.5)$, $(-180^\circ, 0)$, $(-150^\circ, 0.5)$, $(-120^\circ, 0.9)$, $(-90^\circ, 1)$

$(-60^\circ, 0.9)$, $(-30^\circ, 0.5)$, $(0^\circ, 0)$, $(30^\circ, -0.5)$, $(60^\circ, -0.9)$

$(90^\circ, -1)$, $(120^\circ, -0.9)$, $(150^\circ, -0.5)$, $(180^\circ, 0)$, $(210^\circ, 0.5)$

$(240^\circ, 0.9)$, $(270^\circ, 1)$, $(300^\circ, 0.9)$, $(330^\circ, 0.5)$, $(360^\circ, 0)$

Scale:

0.1 = One small square

1 = One big square

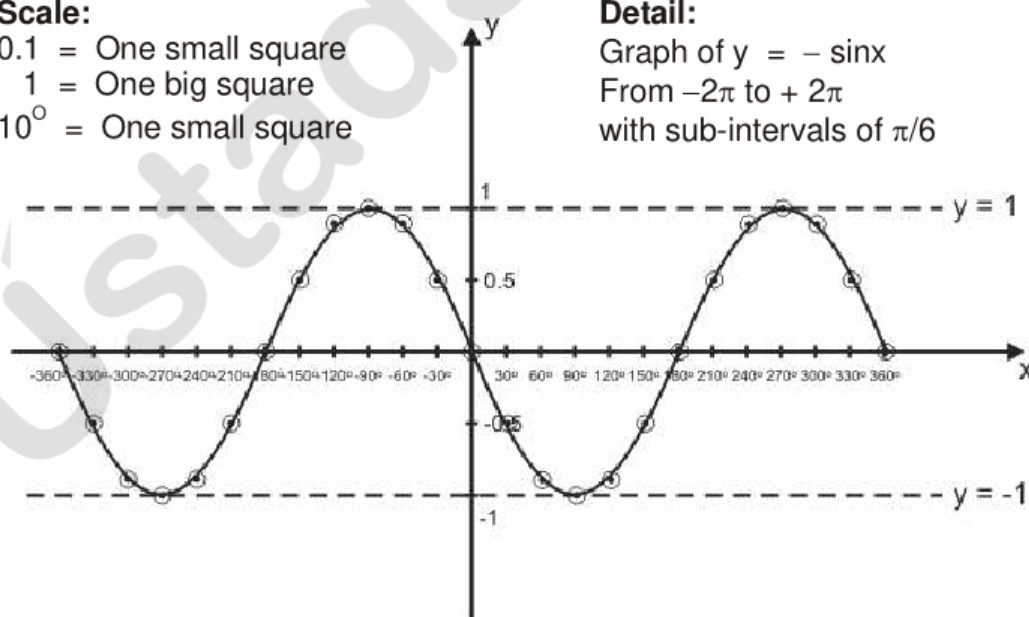
10° = One small square

Detail:

Graph of $y = -\sin x$

From -2π to $+2\pi$

with sub-intervals of $\pi/6$



(ii) $y = 2 \cos x$; $x \in [0, 2\pi]$

Solution:

Given function is $y = 2 \cos x$

We want to draw the graph of $y = 2 \cos x$ from 0 to 360° .

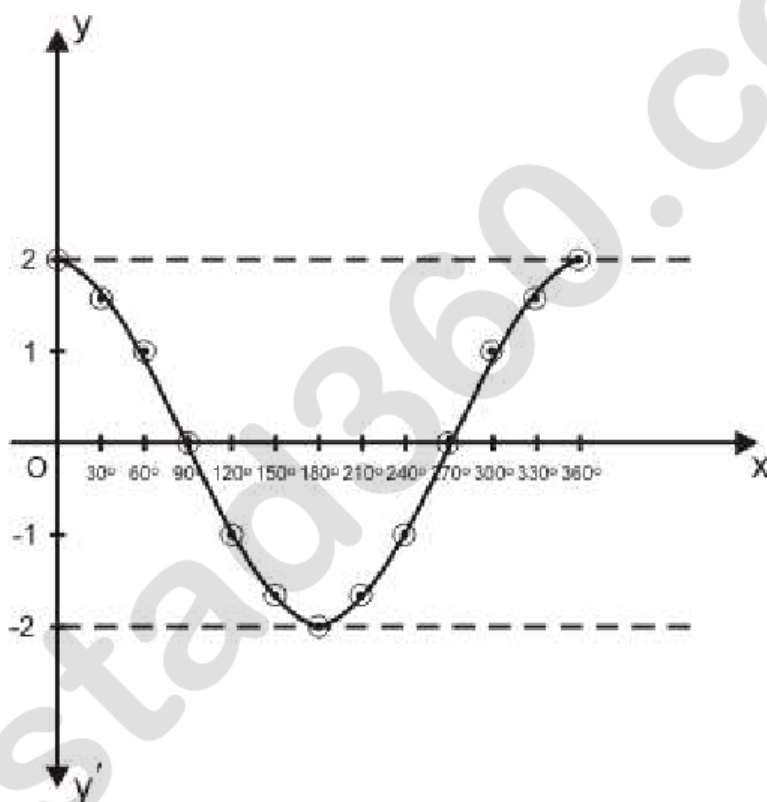
Taking the sub-interval of 30° , we have the following table of values.

x	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
$y = \cos x$	1	0.9	0.5	0	-0.5	-0.9	-1	-0.9	-0.5	0	0.5	0.9	1
$y = 2 \cos x$	2	1.8	1	0	-1	-1.8	-2	-1.8	-1	0	1	1.8	2

Pairs

$(0^\circ, 2), (30^\circ, 1.8), (60^\circ, 1), (90^\circ, 0), (120^\circ, -1), (150^\circ, -1.8), (180^\circ, -2)$

$(210^\circ, -1.8), (240^\circ, -1), (270^\circ, 0), (300^\circ, 1), (330^\circ, 1.8), (360^\circ, 2)$



Scale:

0.1 = One small square

1 = One big square

10° = One small square



(iii) $y = \tan 2x$; $x \in [-\pi, \pi]$

Solution:

Given function is $y = \tan 2x$

We want to draw the graph of $y = \tan 2x$ from -180° to $+180^\circ$.

Taking the sub-interval of 30° , we have the following table of values.

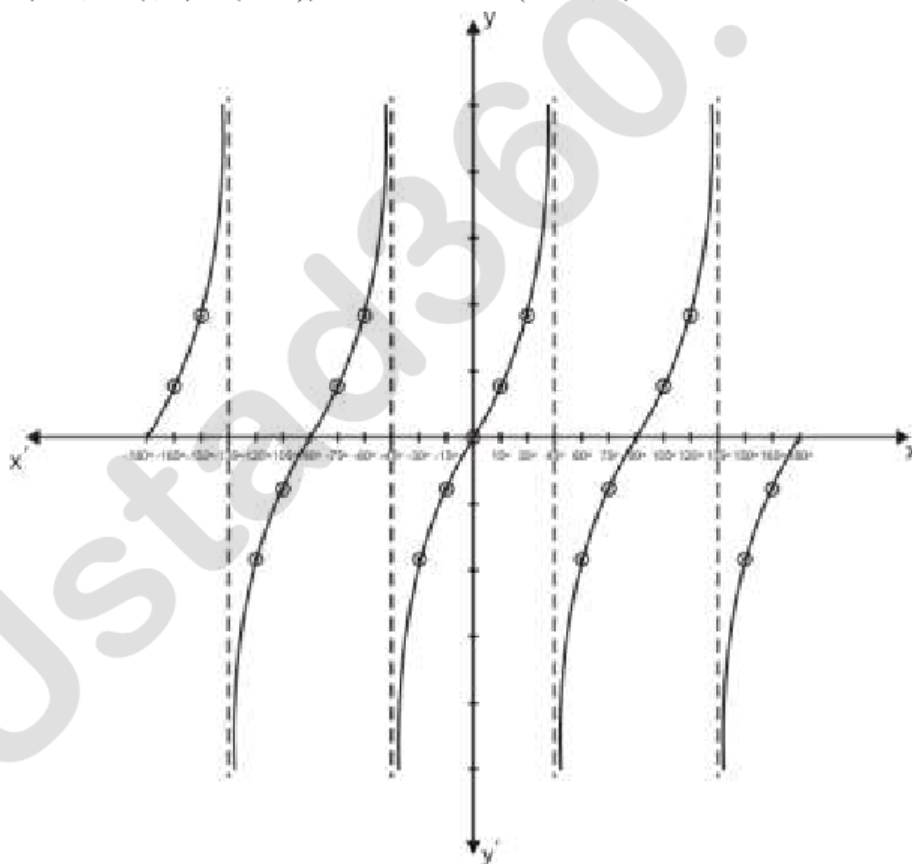
x	-180°	-165°	-150°	-135°	-120°	-105°	-90°	-75°	-60°	-45°	-30°	-15°	0°
2x	-360°	-330°	-300°	-270°	-240°	-210°	-180°	-150°	-125°	-90°	-60°	-30°	0
$y = \tan 2x$	0	0.6	1.73	∞	-1.73	-0.6	0	0.6	1.7	∞	-1.7	-0.6	0

x	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°	180°
2x	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
$y = \tan 2x$	0.6	1.7	∞	-1.7	-0.6	0	0.6	1.7	∞	-1.7	-0.6	0

Pairs

$(-180^\circ, 0), (-165^\circ, 0.6), (-150^\circ, 1.73), \dots, (-15^\circ, -0.6)$

$(0, 0), (15^\circ, 0.6), (30^\circ, 1.7), \dots, (180^\circ, 0)$



Scale:

0.1 = One small square , 1 = One big square , 10° = One small square



(iv) $y = \tan x$; $x \in [-2\pi, 2\pi]$

Solution:

Given function is $y = \tan x$

We want to draw the graph of $y = \tan x$ from -360° to $+360^\circ$.

Taking the sub-interval of 30° , we have the following table of values.

x	-360°	-330°	-300°	-270°	-240°	-210°	-180°	-150°	-120°
$y = \tan x$	0	+0.6	+1.7	∞	-1.7	-0.6	0	0.6	1.7

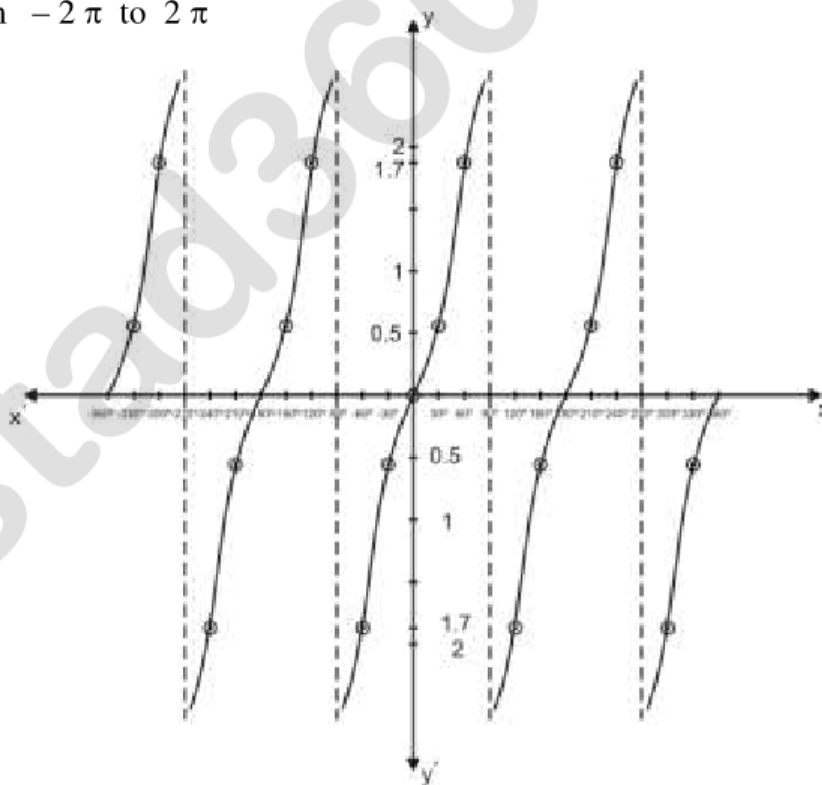
x	-90°	-60°	-30°	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°
$y = \tan x$	$-\infty$	-1.7	-0.6	0	0.6	1.7	$+\infty$	-1.7	-0.6	0	0.6	1.7	$+\infty$	-1.7

x	330°	360°
$y = \tan x$	-0.6	0

Pairs

$(-360^\circ, 0), (-330^\circ, 0.6), (-300^\circ, 1.7), \dots, (360^\circ, 0)$

Graph of $y = \tan x$ from -2π to 2π



Scale:

0.1 = One small square , 1 = One big square , 10° = One small square



(v) $y = \sin \frac{x}{2}; \quad x \in [0, 2\pi]$

Solution:

Given function is $y = \sin \frac{x}{2}$

We want to draw the graph of $y = \sin x$ from 0° to 360° .

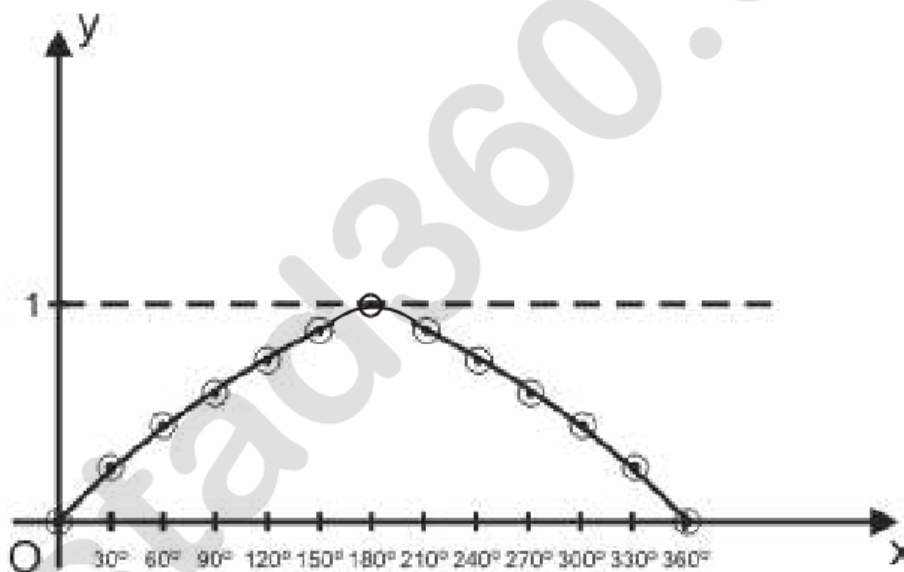
Taking the sub-interval of 30° , we have the following table of values.

x	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
$\frac{x}{2}$	0°	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°	180°
$y = \sin \frac{x}{2}$	0	0.3	0.5	0.7	0.8	0.95	1	0.95	0.8	0.7	0.5	0.3	0

Pairs

$(0^\circ, 0), (30^\circ, 0.3), (60^\circ, 0.5), (90^\circ, 0.7), (120^\circ, 0.8), (150^\circ, 0.95)$

$(180^\circ, 1), (210^\circ, 0.95), (240^\circ, 0.8), (270^\circ, 0.7), (300^\circ, 0.5), (330^\circ, 0.3), (360^\circ, 0)$



Scale:

0.1 = One small square , 1 = One big square , 10° = One small square

(vi) $y = \cos \frac{x}{2}; \quad x \in [-\pi, \pi]$

Solution:

Given function is $y = \cos \frac{x}{2}$



We want to draw the graph of $y = \cos \frac{x}{2}$ from -180° to 180° .

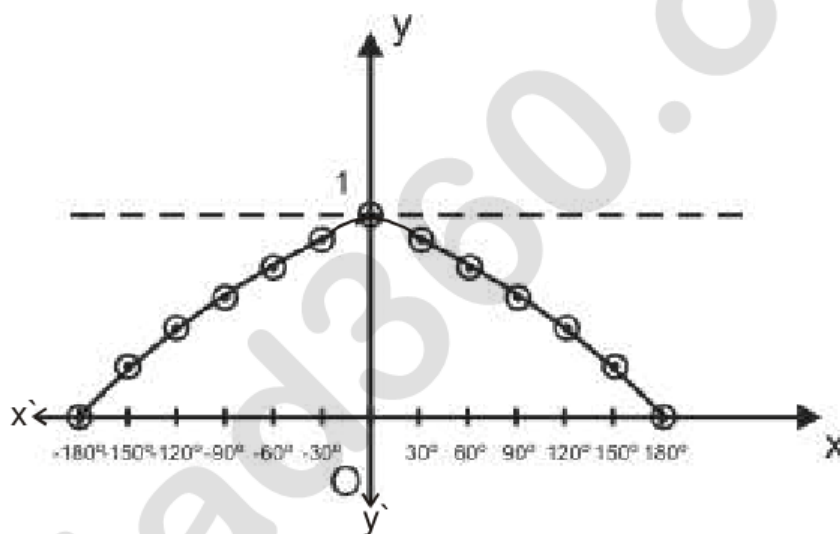
Taking the sub-interval of 30° , we have the following table of values.

$$y = \cos \frac{x}{2}$$

x	-180°	-150°	-120°	-90°	-60°	-30°	0°	30°	60°	90°	120°	150°	180°
$\frac{x}{2}$	-90°	-75°	-60°	-45°	-30°	-15°	0	15°	30°	45°	60°	75°	90°
$y = \cos \frac{x}{2}$	0	0.3	0.5	0.7	0.8	0.95	1	0.95	0.8	0.7	0.5	0.3	0

Pairs

$(-180^\circ, 0)$, $(-150^\circ, 0.3)$, $(-120^\circ, 0.5)$, $(-90^\circ, 0.7)$, $(-60^\circ, 0.8)$
 $(-30^\circ, 0.95)$, $(0^\circ, 1)$, $(30^\circ, 0.95)$, $(60^\circ, 0.8)$, $(90^\circ, 0.7)$
 $(120^\circ, 0.5)$, $(150^\circ, 0.3)$, $(180^\circ, 0)$



Scale:

0.1 = One small square , 1 = One big square , 10° = One small square

Q.2 On the same axes and to the same scale, draw the graphs of the following equations.

(i) $y = \sin x$ and $y = \sin 2x$

Solution:

Given functions are $y = \sin x$ and $y = \sin 2x$

As complete period of $\sin x$ is 2π



So we want to draw the graph of $y = \sin x$ from 0 to 360° .

As complete period of $\sin 2x$ is π

So we want to draw the graph of $y = \sin 2x$ from 0 to 180° .

Taking sub interval of 30° . We have the following tables of values.

$$y = \sin x$$

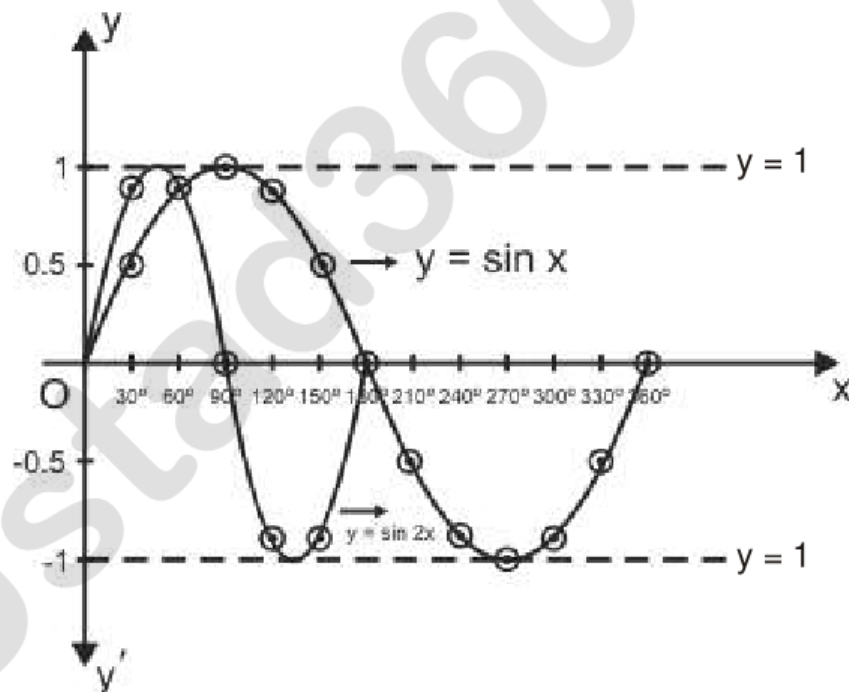
$$y = \cos \frac{x}{2}$$

x	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
$y = \sin x$	0	0.5	0.9	1	0.9	0.5	0	-0.5	-0.9	-1	-0.9	-0.5	0

x	0°	30°	60°	90°	120°	150°	180°
2x	0	60°	120°	180°	240°	300°	360°
$y = \sin 2x$	0	0.8	0.9	0	-0.9	-0.8	0

Pairs

$(0^\circ, 0)$, $(30^\circ, 0.8)$, $(60^\circ, 0.9)$, $(90^\circ, 0)$, $(120^\circ, 0.9)$, $(150^\circ, 0.8)$, $(180^\circ, 0)$



Scale:

$0.1 =$ One small square , $1 =$ One big square , $10^\circ =$ One small square



- (ii) On the same axes and to the same scale, draw the graphs of the following equations for their complete period.

$$y = \cos x \quad \text{and} \quad y = \cos 2x$$

Solution:

Given functions are $y = \cos x$ and $y = \cos 2x$

Firstly, $y = \cos x$, as complete period of $\cos x$ is 2π

So, we want to draw the graph of $y = \cos x$ from 0° to 360° .

Secondly, $y = \cos 2x$, as complete period of $\cos 2x$ is π

So we want to draw the graph of $y = \cos 2x$ from 0 to 180° .

Taking sub-interval of 30° , we have the following tables of values.

$$y = \cos x$$

x	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
$y = \cos x$	1	0.9	0.5	0	-0.5	-0.9	-1	-0.9	-0.5	0	0.5	0.9	1

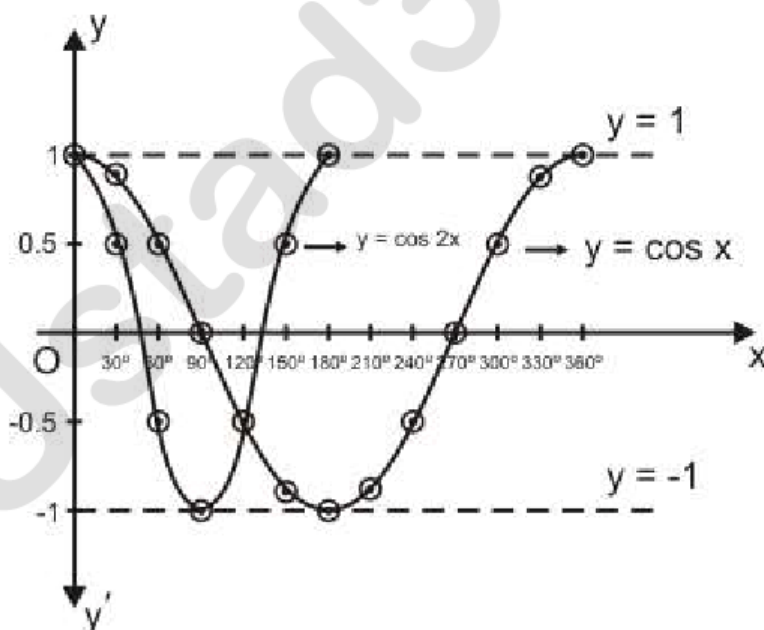
Pairs

$(0^\circ, 1), (30^\circ, 0.9), (60^\circ, 0.5), \dots, (360^\circ, 1)$

x	0°	30°	60°	90°	120°	150°	180°
2x	0	60°	120°	180°	240°	300°	360°
$y = \cos 2x$	1	0.8	-0.5	1	-0.5	0.5	1

Pairs

$(0^\circ, 1), (30^\circ, 0.5), (60^\circ, -0.5), (120^\circ, -0.5), (150^\circ, 0.5), (180^\circ, 1)$



Scale:

$0.1 = \text{One small square}$, $1 = \text{One big square}$, $10^\circ = \text{One small square}$



Q.3 Solve graphically

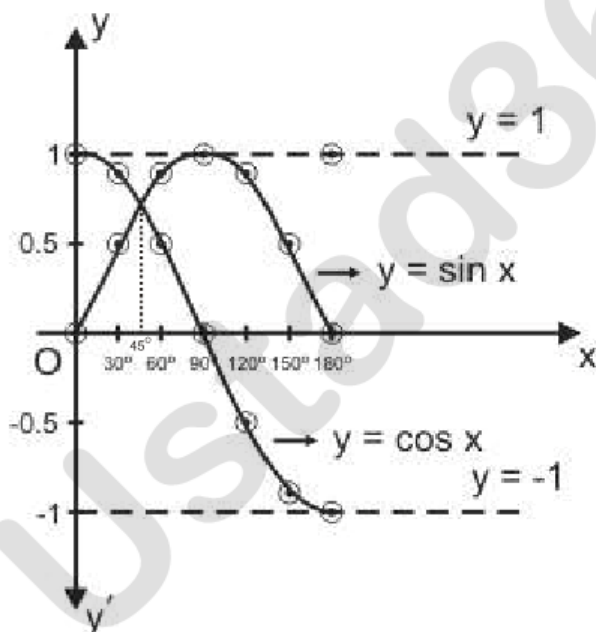
(i) $\sin x = \cos x$; $x \in [0, \pi]$

Solution:Given equation is $\sin x = \cos x$

The solution of this equation will be the point of intersection of the graphs of

 $y = \sin x$ and $y = \cos x$ We want to draw the graph of $y = \sin x$ and $y = \cos x$ from 0 to 180° .Taking the sub-interval of 30° , we have the following table of values:

x	0°	30°	60°	90°	120°	150°	180°
$y = \sin x$	0	0.5	0.9	1	0.9	0.5	0
$y = \cos x$	1	0.9	0.5	0	-0.5	-0.9	-1

The pairs for $y = \sin x$ are $(0^\circ, 0), (30^\circ, 0.5), (60^\circ, 0.9), (90^\circ, 1), (120^\circ, 0.9), (150^\circ, 0.5), (180^\circ, 0)$ **The pairs for $y = \cos x$ are** $(0^\circ, 1), (30^\circ, 0.9), (60^\circ, 0.5), (90^\circ, 0), (120^\circ, -0.5), (150^\circ, -0.9), (180^\circ, -1)$ 

From graph, the point of intersection of $y = \sin x$ and $y = \cos x$ is $x = 45^\circ$.
Thus solution set of equation $\sin x = \cos x$ is $\{\pi/4\}$. Ans.

Scale: $0.1 =$ One small square , $1 =$ One big square , $10^\circ =$ One small square

(ii) Solve graphically

$$\sin x = x; \quad x \in [0, \pi]$$

Solution:

Given equation is $\sin x = x$

The solution of this equation will be the point of intersection of the graphs of $y = \sin x$ and $y = x$

Here we want to draw the graph of $y = \sin x$ and $y = x$ from 0° to 180° .

Taking the sub-interval of 30° , we have the following table of values:

x	0°	30°	60°	90°	120°	150°	180°
$y = \sin x$	0	0.5	0.9	1	0.9	0.5	0
x in radians	0	0.5	1.1	1.6	2.1	2.6	3.1
$y = x$	0	0.5	1.1	1.6	2.1	2.6	3.1

Pairs for $y = \sin x$ are

$(0^\circ, 0), (30^\circ, 0.5), (60^\circ, 0.9), (90^\circ, 1), (120^\circ, 0.9), (150^\circ, 0.5), (180^\circ, 0)$

The pairs for $y = x$ are

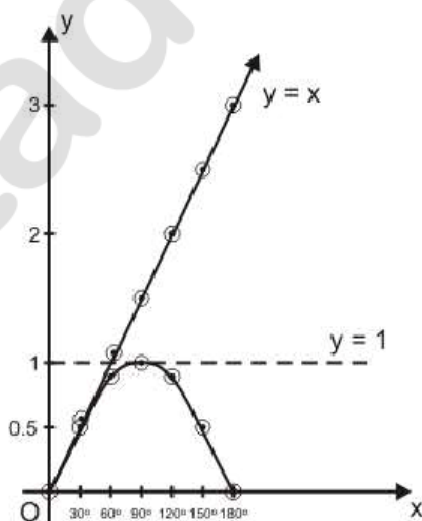
$(0^\circ, 0), (30^\circ, 0.5), (60^\circ, 1.1), (90^\circ, 1.6), (120^\circ, 2.1), (150^\circ, 2.6), (180^\circ, 3.1)$

From graph, the point of intersection of the equations.

$y = \sin x$ and $y = x$ is 0.

So solution set of the equation $\sin x = x$ is $\{0\}$.

Note: To convert degrees into radians, multiply then by $\frac{\pi}{180}$

**Scale:**

0.1 = One small square, 1 = One big square, 10° = One small square

