



CHAPTER

14

SOLUTIONS OF TRIGONOMETRIC EQUATIONS

EXERCISE 14.1

Trigonometric Equations:

The equations containing at least one trigonometric functions are called trigonometric equations.

e.g., $\sin x = \frac{2}{5}$, $\sec x = \tan x$

Q.1 Find the solutions of the following equation which lie in $[0, 2\pi]$

(i) $\sin x = \frac{-\sqrt{3}}{2}$

(ii) $\operatorname{cosec} \theta = 2$ (Gujranwala Board 2005, Lahore Board 2006)

(iii) $\sec x = -2$

(iv) $\cot \theta = \frac{1}{\sqrt{3}}$ (Lahore Board 2010)

Solution:

(i) $\sin x = \frac{-\sqrt{3}}{2}$

$$x = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60^\circ = \frac{\pi}{3}$$

Since $\sin x$ is -ve in III & IV Quadrants with the reference angle $\pi/3$ thus we have

For III-Quadrant

$$x = \pi + \theta$$

$$x = \pi + \frac{\pi}{3}$$

$$x = \frac{4\pi}{3}$$

For IV-Quadrant

$$x = 2\pi - \theta$$

$$x = 2\pi - \frac{\pi}{3}$$

$$x = \frac{5\pi}{3}$$

So thus the required solution is $x = \frac{4\pi}{3}, \frac{5\pi}{3}$



(ii) $\operatorname{cosec} \theta = 2$

$$\frac{1}{\sin \theta} = 2$$

$$\Rightarrow \sin \theta = \frac{1}{2}$$

$$\theta = \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ = \frac{\pi}{6}$$

Since $\sin \theta$ is +ve in I and II Quadrants with reference angle $\frac{\pi}{6}$ Thus

For I-Quadrant

$$x = \frac{\pi}{6},$$

For II-Quadrant

$$x = \pi - \theta$$

$$x = \pi - \frac{\pi}{6}$$

$$x = \frac{5\pi}{6}$$

So thus the required solution is $x = \frac{\pi}{6}, \frac{5\pi}{6}$.

(iii) $\sec x = -2$

$$\frac{1}{\cos x} = -2 \Rightarrow \cos x = -\frac{1}{2}$$

$$x = \cos^{-1}\left(-\frac{1}{2}\right) = 120^\circ = \frac{2\pi}{3}$$

Since $\cos x$ is -ve in II & III Quadrants with reference angle $\frac{\pi}{3}$ thus we have

For II-Quadrant

$$x = \pi - \theta,$$

$$x = \pi - \frac{\pi}{3},$$

$$x = \frac{2\pi}{3},$$

For III-Quadrant

$$x = \pi + \theta$$

$$x = \pi + \frac{\pi}{3}$$

$$x = \frac{4\pi}{3}$$

Thus the required solution is $x = \frac{2\pi}{3}, \frac{4\pi}{3}$

(iv) $\cot \theta = \frac{1}{\sqrt{3}}$

$$\tan \theta = \sqrt{3}$$

$$\theta = \tan^{-1}(\sqrt{3}) = 60^\circ = \frac{\pi}{3}$$

Since $\tan \theta$ is +ve in I & III Quadrants with reference angle $\frac{\pi}{3}$ thus we have.



For I-Quadrant

$$x = \frac{\pi}{3}$$

Thus the required solution is $x = \frac{\pi}{3}, \frac{4\pi}{3}$ Ans.

For III-Quadrant

$$x = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$$

Q.2 Solve the following trigonometric equations:

$$(i) \quad \tan^2 \theta = \frac{1}{3}$$

$$(ii) \quad \operatorname{cosec}^2 \theta = \frac{4}{3}$$

$$(iii) \quad \sec^2 \theta = \frac{4}{3}$$

$$(iv) \quad \cot^2 \theta = \frac{1}{3} \text{ (Lahore Board 2007)}$$

Solution:

$$(i) \quad \tan^2 \theta = \frac{1}{3}$$

$$\tan \theta = \pm \frac{1}{\sqrt{3}}$$

$$\Rightarrow \tan \theta = \frac{1}{\sqrt{3}} \quad \text{and} \quad \tan \theta = -\frac{1}{\sqrt{3}}$$

Since $\tan \theta$ is +ve in I & III Quadrants with reference angle $\frac{\pi}{6}$

Therefore

For I-Quad

$$\theta = \frac{\pi}{6}$$

$$\theta = \frac{\pi}{6} + n\pi,$$

For III-Quad

$$\theta = \pi + \frac{\pi}{6}$$

$$\theta = \frac{7\pi}{6}$$

$$\theta = \frac{7\pi}{6} + n\pi$$

$$\left\{ \frac{\pi}{6} + n\pi \right\} \cup \left\{ \frac{7\pi}{6} + n\pi \right\} \cup \left\{ \frac{5\pi}{6} + n\pi \right\} \cup \left\{ \frac{11\pi}{6} + n\pi \right\}, \forall n \in \mathbb{Z}$$

$$(ii) \quad \operatorname{cosec}^2 \theta = \frac{4}{3}$$

$$\operatorname{cosec} \theta = \pm \frac{2}{\sqrt{3}}$$

$$\operatorname{cosec} \theta = \frac{2}{\sqrt{3}}$$

$$\sin \theta = \frac{\sqrt{3}}{2}$$

Since $\sin \theta$ is +ve in I & II Quadrants with reference angle $\frac{\pi}{3}$

Since $\tan \theta$ is -ve in II & IV Quadrants with reference angle $\frac{\pi}{6}$

For II-Quad

$$\theta = \pi - \frac{\pi}{6}$$

$$\theta = \frac{5\pi}{6}$$

$$\theta = \frac{5\pi}{6} + n\pi$$

For IV-Quad

$$\theta = 2\pi - \frac{\pi}{6}$$

$$\theta = \frac{11\pi}{6}$$

$$\theta = \frac{11\pi}{6} + n\pi \quad \forall n \in \mathbb{Z}$$

$$\theta = \sin^{-1} \left(\frac{\sqrt{3}}{2} \right) = 60^\circ = \frac{\pi}{3}$$

$$\operatorname{cosec} \theta = \frac{-2}{\sqrt{3}}$$

$$\sin \theta = \frac{-\sqrt{3}}{2}$$

Since $\sin \theta$ is -ve in III & IV Quadrants with reference angle $\frac{\pi}{3}$



For I-Quad	For II-Quad	For III-Quad	For IV-Quad
$\theta = \frac{\pi}{3}$	$\theta = \pi - \frac{\pi}{3}$	$\theta = \pi + \frac{\pi}{3}$	$\theta = 2\pi - \frac{\pi}{3}$
$\theta = \frac{\pi}{3} + 2n\pi$	$\theta = \frac{2\pi}{3} + 2n\pi$	$\theta = \frac{4\pi}{3} + 2n\pi$	$\theta = \frac{5\pi}{3} + 2n\pi$

$$\text{S.S} = \left\{ \frac{\pi}{3} + 2n\pi \right\} \cup \left\{ \frac{2\pi}{3} + 2n\pi \right\} \cup \left\{ \frac{4\pi}{3} + 2n\pi \right\} \cup \left\{ \frac{5\pi}{3} + 2n\pi \right\}, \forall n \in \mathbb{Z} \quad \text{Ans.}$$

(iii) $\sec^2 \theta = \frac{4}{3}$ $\theta = \cos^{-1} \left(\frac{\sqrt{3}}{2} \right) = 30^\circ = \frac{\pi}{6}$

$$\sec \theta = \pm \frac{2}{\sqrt{3}}$$

$$\sec \theta = \frac{2}{\sqrt{3}}$$

$$\Rightarrow \cos \theta = \frac{\sqrt{3}}{2}$$

Since $\cos \theta$ is +ve in I & IV Quadrants,

with reference angle $\frac{\pi}{6}$ therefore we have

For I-Quad For IV-Quad

$$\theta = \frac{\pi}{6}, \quad \theta = 2\pi - \frac{\pi}{6}$$

$$\theta = \frac{\pi}{6} + 2n\pi, \quad \theta = \frac{11\pi}{6} + 2n\pi$$

$$\sec \theta = \frac{-2}{\sqrt{3}}$$

$$\cos \theta = \frac{-\sqrt{3}}{2}$$

Since $\cos \theta$ is -ve in II & III Quadrants

with reference angle $\frac{\pi}{6}$ therefore we have

For II-Quad For III-Quad

$$\theta = \pi - \frac{\pi}{6}, \quad \theta = \pi + \frac{\pi}{6}$$

$$\theta = \frac{5\pi}{6} + 2n\pi, \quad \theta = \frac{7\pi}{6} + 2n\pi$$

$$\text{S.S} = \left\{ \frac{\pi}{6} + 2n\pi \right\} \cup \left\{ \frac{4\pi}{6} + 2n\pi \right\} \cup \left\{ \frac{5\pi}{6} + 2n\pi \right\} \cup \left\{ \frac{7\pi}{6} + 2n\pi \right\}, \forall n \in \mathbb{Z} \quad \text{Ans.}$$



$$(iv) \quad \cot^2 \theta = \frac{1}{3}$$

(Lahore Board 2007)

$$\Rightarrow \cot \theta = \pm \frac{1}{\sqrt{3}}$$

$$\tan \theta = \pm \sqrt{3}$$

$$\tan \theta = \sqrt{3}$$

Since $\tan \theta$ is +ve in I & IIIQuadrants with reference angle $\frac{\pi}{3}$

therefore we have

For I-Quad For III-Quad

$$\theta = \frac{\pi}{3}, \quad \theta = \pi + \frac{\pi}{3}$$

$$\theta = \frac{\pi}{3} + n\pi, \quad \theta = \frac{4\pi}{3} + n\pi$$

$$\theta = \tan^{-1}(\sqrt{3}) = 60^\circ = \frac{\pi}{3}$$

$$\tan \theta = -\sqrt{3}$$

Since $\tan \theta$ is -ve in II & IVQuadrants, with reference angle $\frac{\pi}{3}$

therefore we have

For II-Quad For IV-Quad

$$\theta = \pi - \frac{\pi}{3}, \quad \theta = 2\pi - \frac{\pi}{3}$$

$$\theta = \frac{2\pi}{3} + n\pi, \quad \theta = \frac{5\pi}{3} + n\pi$$

$$S.S = \left\{ \frac{\pi}{3} + n\pi \right\} \cup \left\{ \frac{4\pi}{3} + n\pi \right\} \cup \left\{ \frac{2\pi}{3} + n\pi \right\} \cup \left\{ \frac{5\pi}{3} + n\pi \right\}, \forall n \in \mathbb{Z} \text{ Ans.}$$

Q.3 Find the values of θ satisfying the following equations:

$$3 \tan^2 \theta + 2\sqrt{3} \tan \theta + 1 = 0 \quad (\text{Gujranwala Board 2006})$$

Solution:

$$3 \tan^2 \theta + 2\sqrt{3} \tan \theta + 1 = 0$$

$$a = 3, \quad b = 2\sqrt{3}, \quad c = 1 \quad \text{by quadratic formula}$$

$$\begin{aligned} \tan \theta &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-2\sqrt{3} \pm \sqrt{(2\sqrt{3})^2 - 4(3)(1)}}{2(3)} \\ &= \frac{-2\sqrt{3} \pm \sqrt{12 - 12}}{6} = \frac{-2\sqrt{3}}{6} = \frac{-1}{\sqrt{3}} \end{aligned}$$

$$\Rightarrow \tan \theta = -\frac{1}{\sqrt{3}}$$

$$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^\circ = \frac{\pi}{6}$$

Since $\tan \theta$ is -ve in II & IV Quadrants, with reference angle $\frac{\pi}{6}$ therefore we have

For II-Quad**For IV-Quad**

$$\theta = \pi - \frac{\pi}{6}, \quad \theta = 2\pi - \frac{\pi}{6}$$

$$\theta = \frac{5\pi}{6} + n\pi, \quad \theta = \frac{11\pi}{6} + n\pi$$

$$\text{S.S} = \left\{ \frac{5\pi}{6} + n\pi \right\} \cup \left\{ \frac{11\pi}{6} + n\pi \right\}, \forall n \in \mathbb{Z} \text{ Ans.}$$

Q.4 $\tan^2 \theta - \sec \theta - 1 = 0$

Solution:

$$\tan^2 \theta - \sec \theta - 1 = 0$$

$$\sec^2 \theta - 1 - \sec \theta - 1 = 0$$

$$\sec^2 \theta - \sec \theta - 2 = 0$$

$$\sec^2 \theta - 2 \sec \theta + \sec \theta - 2 = 0$$

$$\sec \theta (\sec \theta - 2) + 1 (\sec \theta - 2) = 0$$

$$(\sec \theta - 2) (\sec \theta + 1) = 0$$

$$\sec \theta = 2$$

$$\cos \theta = \frac{1}{2}$$

$$\sec \theta = -1$$

$$\cos \theta = -1$$

$$\Rightarrow \theta = \cos^{-1}(-1)$$

$$\theta = \cos^{-1}(-1)$$

$$\theta = \pi + 2n\pi$$

Since $\cos \theta$ is +ve in I & IV QuadrantsWith reference angle $\frac{\pi}{3}$ **For I-Quad. For IV-Quad.**

$$\theta = \frac{\pi}{3}, \quad \theta = 2\pi - \frac{\pi}{3}$$

$$\theta = \frac{\pi}{3} + 2n\pi, \quad \theta = \frac{5\pi}{3} + 2n\pi$$

$$\text{S.S} = \left\{ \frac{\pi}{3} + 2n\pi \right\} \cup \left\{ \frac{5\pi}{3} + 2n\pi \right\} \cup \{(2n+1)\pi\}, \forall n \in \mathbb{Z}$$

Ans.

Q.5 $2\sin\theta + \cos^2\theta - 1 = 0$

Solution:

$$2\sin\theta + 1 - \sin^2\theta - 1 = 0$$

$$2\sin\theta - \sin^2\theta = 0$$

$$\sin\theta(2 - \sin\theta) = 0$$



$$\Rightarrow \sin\theta = 0, 2 - \sin\theta = 0$$

$$\Rightarrow \begin{array}{l|l} \sin\theta = 0 & \sin\theta = 2 \\ \theta = n\pi & 2 - \sin\theta = 0 \\ & \sin\theta = 0 \end{array}$$

Which is not possible because $-1 \leq \sin\theta \leq 1$

$$S.S = \{n\pi, \forall n \in \mathbb{Z}\}$$

Q.6 $2\sin^2\theta - \sin\theta = 0$

Solution:

$$\begin{array}{l} \sin\theta(2\sin\theta - 1) = 0 \\ \Rightarrow \sin\theta = 0 \text{ and } 2\sin\theta - 1 = 0 \\ \qquad \qquad \qquad 2\sin\theta = 1 \\ \qquad \qquad \qquad \sin\theta = 1/2 \end{array}$$

$$\Rightarrow \begin{array}{l|l} \sin\theta = 0 & \sin\theta = 1/2 \\ \theta = n\pi & \text{Since } \sin\theta \text{ is positive in I and II quadrants with the reference angle } \pi/6. \end{array}$$

I quad. $\theta = \pi/6 + 2n\pi, \forall n \in \mathbb{Z}$	II quad. $\theta = \pi - \pi/6$ $\theta = 5\pi/6$ $\theta = 5\pi/6 + 2n\pi, \forall n \in \mathbb{Z}$
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$$S.S = \{n\pi\} \cup \{\pi/6 + 2n\pi\} \cup \{5\pi/6 + 2n\pi\}, \forall n \in \mathbb{Z}$$

Q.7 $3\cos^2\theta - 2\sqrt{3}\sin\theta\cos\theta - 3\sin^2\theta = 0$

Solution:

$$\begin{aligned} 3\cos^2\theta - 2\sqrt{3}\sin\theta\cos\theta - \sqrt{3}\sin\theta\cos\theta - 3\sin^2\theta &= 0 \\ 3\cos\theta(\cos\theta - \sqrt{3}\sin\theta) + \sqrt{3}\sin\theta(\cos\theta - \sqrt{3}\sin\theta) &= 0 \\ (\cos\theta - \sqrt{3}\sin\theta)(3\cos\theta + \sqrt{3}\sin\theta) &= 0 \end{aligned}$$

$$\cos\theta - \sqrt{3}\sin\theta = 0$$

$$\cos\theta = \sqrt{3}\sin\theta$$

$$\tan\theta = \frac{1}{\sqrt{3}}$$

Since $\tan\theta$ is positive in I to III quadrants with the reference angle $\pi/6$

I quad.

$$\theta = \pi/6 + n\pi$$

III quad

$$\theta = \pi + \pi/6$$

$$\theta = 7\pi/6$$

$$\theta = 7\pi/6 + n\pi$$

$$3\cos\theta + \sqrt{3}\sin\theta = 0$$

$$\sqrt{3}\sin\theta = -3\cos\theta$$

$$\tan\theta = -\sqrt{3}$$

Since $\tan\theta$ is negative in III and IV quadrants with the reference angle $\pi/3$.

I quad.

$$\theta = \pi - \pi/3$$

$$\theta = 2\pi/3$$

$$\theta = 2\pi/3 + n\pi$$

III quad

$$\theta = 2\pi - \pi/3$$

$$\theta = 5\pi/3$$

$$\theta = 5\pi/3 + n\pi$$

$$S.S = \{\pi/6 + n\pi\} \cup \{7\pi/6 + n\pi\} \cup \{2\pi/3 + n\pi\} \cup \{5\pi/3 + n\pi\}, \forall n \in \mathbb{Z}$$



Q.8 Find the values of θ $4 \sin^2 \theta - 8 \cos \theta + 1 = 0$

Solution:

$$4 \sin^2 \theta - 8 \cos \theta + 1 = 0$$

$$4 (1 - \cos^2 \theta) - 8 \cos \theta + 1 = 0$$

$$4 - 4 \cos^2 \theta - 8 \cos \theta + 1 = 0$$

$$4 \cos^2 \theta + 8 \cos \theta - 5 = 0$$

$$4 \cos^2 \theta + 10 \cos \theta - 2 \cos \theta - 5 = 0$$

$$2 \cos \theta (2 \cos \theta + 5) - 1 (2 \cos \theta + 5) = 0$$

$$(2 \cos \theta + 5) (2 \cos \theta - 1) = 0$$

$$2 \cos \theta + 5 = 0$$

$$\cos \theta = \frac{-5}{2}$$

i.e. solution is impossible.

$$2 \cos \theta - 1 = 0$$

$$\cos \theta = \frac{1}{2}$$

$$\theta = \cos^{-1} \left(\frac{1}{2} \right) = 60^\circ = \frac{\pi}{3}$$

Since $\cos \theta$ is +ve in I & IV Quadrants with reference angle $\frac{\pi}{3}$

For I-Quad.

$$\theta = \frac{\pi}{3}$$

$$\theta = \frac{\pi}{3} + 2n\pi,$$

For IV-Quad.

$$\theta = 2\pi - \frac{\pi}{3}$$

$$= \frac{5\pi}{3} + 2n\pi$$

$$\text{S.S} = \left\{ \frac{\pi}{3} + 2n\pi \right\} \cup \left\{ \frac{5\pi}{3} + 2n\pi \right\}, \forall n \in \mathbb{Z} \text{ Ans.}$$

Q.9 Find the solution set of the following equations.

$$\sqrt{3} \tan x - \sec x - 1 = 0$$

(Gujranwala Board 2004)

Solution:

$$\sqrt{3} \tan x = 1 + \sec x$$

$$(\sqrt{3} \tan x)^2 = (1 + \sec x)^2$$

$$3 \tan^2 x = 1 + \sec^2 x + 2 \sec x$$

$$3 (\sec^2 x - 1) = 1 + \sec^2 x + 2 \sec x$$

$$3 \sec^2 x - 3 - 1 - \sec^2 x - 2 \sec x = 0$$

$$2 \sec^2 x - 2 \sec x - 4 = 0$$

$$\sec^2 x - \sec x - 2 = 0$$



$$\sec^2 x - 2 \sec x + \sec x - 2 = 0$$

$$\sec x (\sec x - 2) + 1 (\sec x - 2) = 0$$

$$(\sec x - 2)(\sec x + 1) = 0$$

$$\sec x - 2 = 0$$

$$\sec x = 2$$

$$\Rightarrow \cos x = \frac{1}{2}$$

$$\sec x + 1 = 0$$

$$\sec x = -1$$

$$\cos x = -1$$

$$x = \cos^{-1}(-1)$$

$$x = \pi$$

$$x = \pi + 2n\pi, \forall n \in \mathbb{Z}$$

Since $\cos x$ is +ve in I & IV Quadrant

with reference angle $\frac{\pi}{3}$

For I-Quad. For II-Quad.

$$x = \frac{\pi}{3}, \quad x = 2\pi - \frac{\pi}{3}$$

$$x = \frac{\pi}{3} + 2n\pi, \quad x = \frac{5\pi}{3} + 2n\pi$$

Solution set is $\left\{ \frac{\pi}{3} + 2n\pi \right\} \cup \left\{ \frac{5\pi}{3} + 2n\pi \right\} \cup \{ \pi + 2n\pi \}, n \in \mathbb{Z}$

Q.10 $\cos 2x = \sin 3x$

Solution:

$$\cos 2x = \sin 3x$$

$$\cos^2 x - \sin^2 x = 3 \sin x - 4 \sin^3 x$$

$$1 - \sin^2 x - \sin^2 x - 3 \sin x + 4 \sin^3 x = 0$$

$$4 \sin^3 x - 2 \sin^2 x - 3 \sin x + 1 = 0$$

The above equation is satisfied by $\sin x = 1$

By synthetic division, we have

4	-2	-3	1	
1	4	2	-1	
4	2	-1	0	

$$4 \sin^2 x + 2 \sin x - 1 = 0$$

Here $a = 4, b = 2, c = -1$



By quadratic formula

$$\sin x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\sin x = \frac{-2 \pm \sqrt{4 - 4(4)(-1)}}{2(4)} = \frac{-2 \pm \sqrt{4 + 16}}{8}$$

$$\sin x = \frac{-2 \pm \sqrt{20}}{8}$$

$$x = \sin^{-1}\left(\frac{\sqrt{20} - 2}{8}\right)$$

$$x = 18^\circ = \frac{\pi}{10}$$

Since $\sin x$ is +ve in the I and II Quadrants
with reference angle $\frac{\pi}{10}$

For I-Quad. For II-Quad.

$$x = \frac{\pi}{10}, \quad x = \pi - \frac{\pi}{10}$$

$$x = \frac{\pi}{10} + 2n\pi, \quad x = \frac{9\pi}{10} + 2n\pi$$

Also $\sin x = 1$

$$x = \frac{\pi}{2} + 2n\pi, \quad \forall n \in \mathbb{Z}$$

Hence solution set is

$$\left\{\frac{\pi}{2} + 2n\pi\right\} \cup \left\{\frac{\pi}{10} + 2n\pi\right\} \cup \left\{\frac{9\pi}{10} + 2n\pi\right\} \cup \left\{\frac{13\pi}{10} + 2n\pi\right\} \cup \left\{\frac{17\pi}{10} + 2n\pi\right\} \quad n \in \mathbb{Z} \quad \text{Ans.}$$

Q.11 $\sec 3\theta = \sec \theta$

Solution:

$$\sec 3\theta = \sec \theta$$

$$\frac{1}{\cos 3\theta} = \frac{1}{\cos \theta}$$

$$\cos 3\theta = \cos \theta$$

$$\cos 3\theta - \cos \theta = 0$$

$$\sin x = \frac{-\sqrt{20} - 2}{8} = -\left(\frac{\sqrt{20} + 2}{8}\right)$$

$$x = \sin^{-1}\left(\frac{\sqrt{20} + 2}{8}\right)$$

$$x = 54^\circ = \frac{3\pi}{10}$$

Since $\sin x$ is -ve in III & IV Quadrants

with reference angle $\frac{3\pi}{10}$

For III-Quad.

$$x = \pi + \frac{3\pi}{10},$$

$$x = \frac{13\pi}{10} + 2n\pi,$$

For IV-Quad.

$$x = 2\pi - \frac{3\pi}{10}$$

$$x = \frac{17\pi}{10} + 2n\pi$$



$$-2 \sin\left(\frac{3\theta + \theta}{2}\right) \sin\left(\frac{3\theta - \theta}{2}\right) = 0$$

$$-2 \sin 2\theta \sin \theta = 0$$

$$\Rightarrow \sin 2\theta \sin \theta = 0$$

$$\Rightarrow \sin 2\theta = 0$$

$$\Rightarrow 2\theta = n\pi$$

$$\Rightarrow \theta = \frac{n\pi}{2}$$

$$\sin \theta = 0$$

$$\Rightarrow \theta = n\pi$$

Solution set is $\left\{\frac{n\pi}{2}\right\} \cup \{n\pi\}$, $n \in \mathbb{Z}$ Ans.

Q.12 $\tan 2\theta + \cot \theta = 0$

Solution:

$$\tan 2\theta = -\cot \theta$$

$$\frac{\sin 2\theta}{\cos 2\theta} = -\frac{\cos \theta}{\sin \theta}$$

$$\sin 2\theta \sin \theta = -\cos 2\theta \cos \theta$$

$$\sin 2\theta \sin \theta + \cos 2\theta \cos \theta = 0$$

$$\Rightarrow \cos (2\theta - \theta) = 0$$

$$\Rightarrow \cos \theta = 0$$

$$\Rightarrow \theta = (2n+1)\frac{\pi}{2}$$

Solution set is $\left\{(2n+1)\frac{\pi}{2}\right\}$, $n \in \mathbb{Z}$

Q.13 $\sin 2x + \sin x = 0$

Solution:

$$\sin 2x + \sin x = 0$$

$$2 \sin x \cos x + \sin x = 0$$

$$\sin x (2 \cos x + 1) = 0$$

$$\sin x = 0$$

$$x = n\pi$$

$$2 \cos x + 1 = 0$$

$$\cos x = -\frac{1}{2}$$

As $\cos x$ is +ve in I & IV Quadrants

with reference angle $\frac{\pi}{3}$

For I-Quad.

$$x = \frac{\pi}{3}$$

$$x = \frac{\pi}{3} + 2n\pi,$$

For II-Quad.

$$x = 2\pi - \frac{\pi}{3}$$

$$x = \frac{5\pi}{3} + 2n\pi$$



Therefore solution set is

$$\{n\pi\} \cup \left\{\frac{\pi}{3} + 2n\pi\right\} \cup \left\{\frac{5\pi}{3} + 2n\pi\right\}, \quad n \in \mathbb{Z}$$

Q.14 $\sin 4x - \sin 2x = \cos 3x$

Solution:

$$\sin 4x - \sin 2x = \cos 3x$$

$$2 \cos \left(\frac{4x+2x}{2}\right) \sin \left(\frac{4x-2x}{2}\right) = \cos 3x$$

$$2 \cos 3x \sin x - \cos 3x = 0$$

$$\cos 3x [2 \sin x - 1] = 0$$

$$\cos 3x = 0$$

$$\Rightarrow 3x = (2n+1)\frac{\pi}{2}$$

$$\Rightarrow x = (2n+1)\frac{\pi}{6}$$

$$2 \sin x - 1 = 0$$

$$\Rightarrow 2 \sin x = 1$$

$$\Rightarrow \sin x = \frac{1}{2} \quad \boxed{x = \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ = \frac{\pi}{6}}$$

Since $\sin x$ is +ve in I and II Quadrants

with reference angle $\frac{\pi}{6}$ so

For I-Quad.

$$x = \frac{\pi}{6}$$

$$x = \frac{\pi}{6} + 2n\pi,$$

For II-Quad.

$$x = \pi - \frac{\pi}{6}$$

$$x = \frac{5\pi}{6} + 2n\pi, \quad \forall n \in \mathbb{Z}$$

Hence solution set is

$$\left\{(2n+1)\frac{\pi}{6}\right\} \cup \left\{\frac{\pi}{6} + 2n\pi\right\} \cup \left\{\frac{5\pi}{6} + 2n\pi\right\}, \quad n \in \mathbb{Z}$$

Ans.

Q.15 $\sin x + \cos 3x = \cos 5x$

Solution:

$$\sin x + \cos 3x = \cos 5x$$

$$\sin x = \cos 5x - \cos 3x$$

$$\sin x = -2 \sin \left(\frac{5x+3x}{2}\right) \sin \left(\frac{5x-3x}{2}\right)$$

$$= -2 \sin 4x \sin x$$



$$\sin x + 2 \sin 4x \sin x = 0$$

$$\sin x (1 + 2 \sin 4x) = 0$$

$$\sin x = 0$$

$$x = n\pi$$

$$\sin 4x = -\frac{1}{2} \Rightarrow 4x = \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ = \frac{\pi}{6}$$

since $\sin x$ is -ve in III & IV Quadrants

with reference angle $\frac{\pi}{6}$

For III-Quad.

For II-Quad.

$$4x = \pi + \frac{\pi}{6}, \quad 4x = 2\pi - \frac{\pi}{6}$$

$$4x = \frac{7\pi}{6} + 2n\pi, \quad 4x = \frac{11\pi}{6} + 2n\pi$$

$$x = \frac{7\pi}{24} + \frac{n\pi}{2}, \quad x = \frac{11\pi}{24} + \frac{n\pi}{2}$$

Hence solution set $\{n\pi\} \cup \left\{\frac{7\pi}{24} + \frac{n\pi}{2}\right\} \cup \left\{\frac{11\pi}{24} + \frac{n\pi}{2}\right\}, \quad n \in \mathbb{Z}$ Ans.

Q.16 $\sin 3x + \sin 2x + \sin x = 0$

Solution:

$$\sin 3x + \sin 2x + \sin x = 0$$

$$\sin 3x + \sin x + \sin 2x = 0$$

$$2 \cos\left(\frac{3x-x}{2}\right) \sin\left(\frac{3x+x}{2}\right) + \sin 2x = 0$$

$$2 \sin 2x \cos x + \sin 2x = 0$$

$$\sin 2x (2 \cos x + 1) = 0$$

$$\sin 2x = 0$$

$$2x = n\pi$$

$$x = \frac{n\pi}{2}$$

$$2 \cos x + 1 = 0$$

$$x = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ = \frac{\pi}{3}$$

$$\cos x = -\frac{1}{2}$$

Since $\cos$ is -ve in II & III Quadrants

with reference angle $\frac{\pi}{3}$

For II-Quad.

For III-Quad.

$$x = \pi - \frac{\pi}{3},$$

$$x = \pi + \frac{\pi}{3}$$

$$x = \frac{2\pi}{3} + 2n\pi,$$

$$x = \frac{4\pi}{3} + 2n\pi, \quad \forall n \in \mathbb{Z}$$

Solution set is $\left\{\frac{n\pi}{2}\right\} \cup \left\{\frac{2\pi}{3} + 2n\pi\right\} \cup \left\{\frac{4\pi}{3} + 2n\pi\right\}, \quad n \in \mathbb{Z}$

Ans.



Q.17 $\sin 7x - \sin x = \sin 3x$ **Solution:**

$$\sin 7x - \sin x = \sin 3x$$

$$2 \cos \left(\frac{7x+x}{2} \right) \sin \left(\frac{7x-x}{2} \right) = \sin 3x$$

$$2 \cos 4x \sin 3x = \sin 3x$$

$$2 \cos 4x \sin 3x - \sin 3x = 0$$

$$\sin 3x (2 \cos 4x - 1) = 0$$

$$\Rightarrow \sin 3x = 0$$

$$3x = n\pi$$

$$x = \frac{n\pi}{3}$$

$$2 \cos 4x - 1 = 0$$

$$2 \cos 4x = 1$$

$$\cos 4x = \frac{1}{2}$$

$$4x = \cos^{-1} \left(\frac{1}{2} \right) = 60^\circ = \frac{\pi}{3}$$

Since $\cos x$ is +ve in I & IV Quadrants.

with reference angle $\frac{\pi}{3}$

For I-Quad.

For IV-Quad.

$$4x = \frac{\pi}{3}$$

$$4x = 2\pi - \frac{\pi}{3}$$

$$4x = \frac{5\pi}{3}$$

$$4x = \frac{\pi}{3} + 2n\pi$$

$$4x = \frac{5\pi}{3} + 2n\pi$$

$$x = \frac{\pi}{12} + \frac{n\pi}{2}$$

$$x = \frac{5\pi}{12} + \frac{n\pi}{2}$$

Therefore solution set is $\left\{ \frac{n\pi}{3} \right\} \cup \left\{ \frac{\pi}{12} + \frac{n\pi}{2} \right\} \cup \left\{ \frac{5\pi}{12} + \frac{n\pi}{2} \right\} \quad n \in \mathbb{Z} \quad \text{Ans.}$

Q.18 $\sin x + \sin 3x + \sin 5x = 0$ **Solution:**

$$\sin 5x + \sin x + \sin 3x = 0$$

$$2 \sin \left(\frac{5x+x}{2} \right) \cos \left(\frac{5x-x}{2} \right) + \sin 3x = 0$$



$$2 \sin 3x \cos 2x + \sin 3x = 0$$

$$\sin 3x (2 \cos 2x + 1) = 0$$

$$\sin 3x = 0$$

$$3x = n\pi$$

$$x = \frac{n\pi}{3}$$

$$2 \cos 2x + 1 = 0$$

$$\cos 2x = \frac{-1}{2}$$

$$2x = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ = \frac{\pi}{3}$$

Since $\cos x$ is -ve in II & III Quadrants,

with reference angle $\frac{\pi}{3}$ so

For I-Quad.

For III-Quad.

$$2x = \pi - \frac{\pi}{3}$$

$$2x = \pi + \frac{\pi}{3}$$

$$2x = \frac{2\pi}{3}$$

$$2x = \frac{4\pi}{3}$$

$$2x = \frac{2\pi}{3} + 2n\pi$$

$$2x = \frac{4\pi}{3} + 2n\pi$$

$$x = \frac{\pi}{3} + \frac{2n\pi}{2}$$

$$x = \frac{2\pi}{3} + \frac{2n\pi}{2}$$

$$x = \frac{\pi}{3} + n\pi$$

$$x = \frac{2\pi}{3} + n\pi$$

Hence solution set is

$$\left\{\frac{n\pi}{3}\right\} \cup \left\{\frac{\pi}{3} + \frac{2n\pi}{3}\right\} \cup \left\{\frac{2\pi}{3} + \frac{2n\pi}{3}\right\}, \quad n \in \mathbb{Z} \quad \text{Ans.}$$

Q.19 $\sin \theta + \sin 3\theta + \sin 5\theta + \sin 7\theta = 0$

Solution:

$$[\sin 7\theta + \sin \theta] + [\sin 5\theta + \sin 3\theta] = 0$$

$$\left[2 \sin\left(\frac{7\theta + \theta}{2}\right) \cos\left(\frac{7\theta - \theta}{2}\right)\right] + \left[2 \sin\left(\frac{5\theta + 3\theta}{2}\right) \cos\left(\frac{5\theta - 3\theta}{2}\right)\right] = 0$$

$$2 \sin 4\theta \cos 3\theta + 2 \sin 4\theta \cos \theta = 0$$

$$2 \sin 4\theta (\cos 3\theta + \cos \theta) = 0$$

$$2 \sin 4\theta \left[2 \cos\left(\frac{3\theta + \theta}{2}\right) \cos\left(\frac{3\theta - \theta}{2}\right)\right] = 0$$

$$2 \times 2 \sin 4\theta (\cos 2\theta \cos \theta) = 0$$

$$\sin 4\theta \cos 2\theta \cos \theta = 0$$



$$\Rightarrow \sin 4\theta = 0 \quad \left| \begin{array}{l} \cos 2\theta = 0 \\ 2\theta = (2n+1)\frac{\pi}{2} \\ \theta = (2n+1)\frac{\pi}{4} \end{array} \right| \quad \left| \begin{array}{l} \cos \theta = 0 \\ \theta = (2n+1)\frac{\pi}{2} \end{array} \right|$$

$$4\theta = n\pi$$

$$\theta = \frac{n\pi}{4}$$

Hence solutions set is

$$\left\{ \frac{n\pi}{4} \right\} \cup \left\{ (2n+1)\frac{\pi}{4} \right\} \cup \left\{ (2n+1)\frac{\pi}{2} \right\}, n \in \mathbb{Z} \quad \text{Ans.}$$

OR

$$\left\{ \frac{n\pi}{4} \right\} \cup \left\{ \frac{\pi}{4} + \frac{n\pi}{2} \right\} \cup \left\{ \frac{\pi}{2} + n\pi \right\}, n \in \mathbb{Z}$$

Q.20 $\cos \theta + \cos 3\theta + \cos 5\theta + \cos 7\theta = 0$

Solution:

$$[\cos 7\theta + \cos \theta] + [\cos 5\theta + \cos 3\theta] = 0$$

$$2 \cos \left(\frac{7\theta + \theta}{2} \right) \cos \left(\frac{7\theta - \theta}{2} \right) + 2 \cos \left(\frac{5\theta + 3\theta}{2} \right) \cos \left(\frac{5\theta - 3\theta}{2} \right) = 0$$

$$2 \cos 4\theta \cos 3\theta + 2 \cos 4\theta \cos \theta = 0$$

$$2 \cos 4\theta (\cos 3\theta + \cos \theta) = 0$$

$$2 \cos 4\theta \left[2 \cos \left(\frac{3\theta + \theta}{2} \right) \cos \left(\frac{3\theta - \theta}{2} \right) \right] = 0$$

$$2 \times 2 \cos 4\theta (\cos 2\theta \cos \theta) = 0$$

$$\Rightarrow \cos 4\theta = 0 \quad \left| \begin{array}{l} \cos 2\theta = 0 \\ 2\theta = (2n+1)\frac{\pi}{2} \\ \theta = (2n+1)\frac{\pi}{4} \end{array} \right| \quad \left| \begin{array}{l} \cos \theta = 0 \\ \theta = (2n+1)\frac{\pi}{2} \end{array} \right|$$

$$4\theta = (2n+1)\frac{\pi}{2}$$

$$\theta = (2n+1)\frac{\pi}{8}$$

$$\text{Hence solution set is } \left\{ (2n+1)\frac{\pi}{8} \right\} \cup \left\{ (2n+1)\frac{\pi}{4} \right\} \cup \left\{ (2n+1)\frac{\pi}{2} \right\}, n \in \mathbb{Z}$$

OR

$$\text{S.S} = \left\{ \frac{\pi}{8} + \frac{n\pi}{4} \right\} \cup \left\{ \frac{\pi}{4} + \frac{n\pi}{2} \right\} \cup \left\{ \frac{\pi}{2} + n\pi \right\}, n \in \mathbb{Z}$$

